BERNOULLI NUMBERS AND BESSEL POLYNOMIALS

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1. Introduction. Put [5, Chapter 2]

$$\frac{2}{e^x + 1} = \sum_{n=0}^{\infty} \frac{C_n}{n!} \left(\frac{x}{2}\right)^n, \tag{1.1}$$

$$\frac{x}{e^x - 1} = \sum_{n=0}^{\infty} B_n \frac{x^n}{n!}, \tag{1.2}$$

so that the B_n are the Bernoulli numbers in the even suffix notation and

$$C_{n-1} = 2^n(1 - 2^n) \frac{B_n}{n}.$$

In the present paper we prove the following formulas which seem to be new.

$$\sum_{k=0}^{n} \frac{(2n + 2k)!}{(2k)!(2n - 2k)!2^{2k}} \frac{C_{2m+2k+1}}{(2m + 2k + 1)!} = -\delta_{mn} \frac{2^{2n}(2n)!}{(4n + 1)!} \qquad (0 \leq m \leq n), \tag{1.3}$$

$$\sum_{k=0}^{n} \frac{(2n + 2k + 2)!}{(2k)!(2n - 2k)!2^{k+1}} \frac{C_{2m+2k+3}}{(2m + 2k + 3)!} = \delta_{mn} \frac{2^{2n+1}(2n + 1)!}{(4n + 3)!} \qquad (0 \leq m \leq n), \tag{1.4}$$

$$\sum_{k=0}^{n} \frac{(2n + 2k + 1)!}{(2k)!(2n - 2k + 1)!} \frac{B_{2m+2k+2}}{(2m + 2k + 2)!} = \delta_{mn} \frac{(2n + 1)!}{2\cdot(4n + 3)!} \qquad (0 \leq m \leq n), \tag{1.5}$$

$$\sum_{k=0}^{n} \frac{(2n + 2k + 1)!}{(2k + 1)!(2n - 2k + 1)!} \frac{B_{2m+2k+4}}{(2m + 2k + 4)!} = -\delta_{mn} \frac{(2n + 2)!}{2\cdot(4n + 5)!} \qquad (0 \leq m \leq n). \tag{1.6}$$

Next let

$$y_n(x) = \sum_{k=0}^{n} \frac{(n + k)!}{k!(n - k)!} \left(\frac{z}{2}\right)^k \tag{1.7}$$

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