SERIES-PARALLEL GRAPHS AND LATTICES

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1. **Introduction.** Motivated by the idea of series-parallel networks a concept of series-parallel graph is formulated (4.3). If one regards a "bridge" as a two-terminal network characterized by the existence of an element through which current may pass in either of two directions (depending on the network parameters), the question whether all non-series-parallel two-terminal networks are bridge networks suggests itself. (See [2], especially the footnote on p. 84 which gives the two alternative definitions of series-parallel proved equivalent (4.12) in this paper.) The converse question is readily answered in the affirmative. The question itself may be answered in the affirmative provided that the class of non-series-parallel networks is restricted to the "sensible" ones. A network (of resistors) which is not "sensible" contains elements which do not influence the behavior of the network. "Non-sensible" networks include all topologically disconnected ones as well as some which are connected, for example, Figure 0. The mathematical formulation of "sensible" as applied to graphs

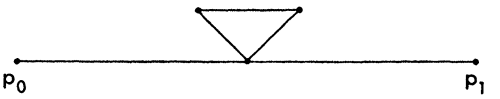

Fig. 0

is the concept of "admissible" graph (4.1) while the mathematical formulation of "bridge" is a two-terminal admissible graph whose link relation (2.2) is not asymmetric. The question raised is answered by the main theorem (4.12): an admissible graph is series-parallel if and only if its link relation is asymmetric. (It has been brought to the attention of the authors that a similar result (unpublished) has been obtained independently by E. F. Moore.)

§2 is devoted to demonstrating that associated with a graph whose link relation L is asymmetric, is a lattice. This lattice has special properties which are embodied in the concept "series-parallel" lattice. §3 studies lattices of this kind in abstracto. In §4 the concept of graph "refinement" is introduced and is used to effect a transition between lattice-theoretic and graph-theoretic considerations. The properties of series-parallel lattices are then used to obtain the main result concerning graphs (4.12). The basic role of series-parallel

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