DIRICHLET MULTIPLICATION IN LATTICE POINT PROBLEMS

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E. Landau [3] has given a theorem on the convergence of the Dirichlet product which can be stated in terms of summatory functions as follows. We assume (a_n) and (b_n) are complex sequences and (ξ_n) is a strictly increasing, unbounded positive sequence. If (η_n) is the strictly increasing sequence of the distinct values of $\xi_\mu \xi_\nu$, $\mu \geq 1$, $\nu \geq 1$, and

$$c_n = \sum_{\xi_\mu \xi_\nu = \eta_n} a_\mu b_\nu$$

we have

THEOREM 1. *Given non-negative real numbers α, β, ρ, τ with $\alpha \leq \rho$, $\beta \leq \tau$, $\rho \geq \beta$, $\tau \geq \alpha$, $\rho + \tau - \alpha - \beta > 0$, if for each $\epsilon > 0$*

$$A(x) = \sum_{\xi_n < x} a_n = O(x^{\alpha + \epsilon}), \qquad \sum_{\xi_n < x} | a_n | = O(x^{\rho + \epsilon}),$$

$$B(x) = \sum_{\xi_n < x} b_n = O(x^{\beta + \epsilon}), \qquad \sum_{\xi_n < x} | b_n | = O(x^{\tau + \epsilon})$$

for all $x \geq 1$, then for each $\epsilon > 0$,

$$C(x) = \sum_{\eta_n < x} c_n = O(x^{\omega + \epsilon})$$

with

$$\omega = \frac{\rho\tau - \alpha\beta}{\rho + \tau - \alpha - \beta}.$$

(We have taken the liberty to change Landau's notation.)

In this paper we shall give in Theorem 2 a generalization of Theorem 1 for the case where $A(x)$ and $B(x)$ are representable in the form

$$\sum_{\mu=1}^{h} x^{\alpha_\mu} P_\mu(\log x) + O(x^\alpha \log^l (x + 1)) \tag{1}$$

where the α_μ are complex numbers and the P_μ are polynomial functions. Thus we are able to summarize a method of attack on a certain class of lattice point problems.

We shall not need to assume that the functions A and B are summatory functions, for we can just as well define C as the Stieltjes resultant of A and B; i.e.

$$C(x) = \int_1^x A(x/u)\, dB(u) \tag{2}$$

Received January 30, 1958.