A COMPARISON THEOREM FOR STURMIAN OSCILLATION NUMBERS OF LINEAR SYSTEMS OF SECOND ORDER.

By Aurel Wintner

1. The criterion to be isolated in this note suggested itself by the comparison theorems of Morse [3] on the self-adjoint Jacobi systems in calculus of variations, as applied by Myers [4] to conjugate points on geodesics in a Riemannian space with a positive definite curvature tensor. For the sake of simplicity, the linear differential system will be assumed in the form

$$x'' + F(t)x = 0 \tag{1}$$

(x is a vector), to be compared with the scalar differential equation

$$u'' + f(t)u = 0, \tag{2}$$

in which $f(t)$ denotes the greatest eigenvalue of the Hermitian component of the coefficient matrix $F(t)$ of (1).

The system (1) will not be assumed to be self-adjoint. The comparison theorem will follow by an appropriate combination of two simple devices I used before to other ends [5]. The result is likely to have applications in the direction of Lusternik and Schnirelmann [2] and the general topological theory of Morse.

2. The notations will be as follows:

On a t-interval Θ of finite or of infinite length, let $F = F(t)$ be an n by n matrix which is a continuous function of t. Since n can be replaced by $2n$, there is no loss of generality in assuming that the n^2 elements of F are real-valued. Correspondingly, the n components of the vector x on which F operates will be confined to the real field. Actually, it will be clear from the proof that nothing is changed if x is a vector in a Hilbert space on which F is a bounded operator (provided that "greatest value contained in the spectrum" is read in place of "greatest eigenvalue").

For a fixed t (on Θ), let $2H$ be the sum, and $2K$ the difference, of F and its transposed matrix; in other words, put $F = H + K$, where H is symmetric and K is skew-symmetric. Then, if $G(x, y)$ is the bilinear form belonging to a matrix G ($= F, H, K$), the form $K(x, x)$ vanishes identically, and so, if a dot denotes scalar multiplication,

$$x \cdot Fx = F(x, x) = H(x, x) = x \cdot Hx \leq f \mid x \mid^2, \tag{3}$$

where f is the greatest eigenvalue of H. Here t is fixed; if it varies on Θ, then $H = H(t)$ and $f = f(t)$ are continuous functions, since $F = F(t)$ is.

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