THE CONTINUOUS TRANSFORMATION RING OF BIORTHOGONAL BASES SPACES

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1. **Introduction.** This note is concerned with biorthogonal bases spaces. These are dual vector spaces M and N over a division ring D[2; 15] such that there exist equipotent bases $\{x_i\}$ for M and $\{y_i\}$ for N with $(x_i, f_j) = \delta_{ij}$. We shall refer to the basis $\{x_i\}$ as the good basis. Some of the interest in biorthogonal bases spaces stems from the following theorem proved by Mackey in [7; 171]: if $[M: D] = [N: D] = \aleph_0$, then M and N are biorthogonal bases spaces.

Throughout this paper M and N will be assumed to be biorthogonal bases spaces. We will let $L(M, N)$ denote the ring of all continuous linear transformations on M and $F(M, N)$ denote the ring of finite-valued linear transformations in $L(M, N)$. Continuous means continuous in the N-topology of M. A transformation a on M is continuous in this topology if and only if there exists a transformation a' on N such that $(xa, f) = (x, a'f)$ for all $x \in M$ and all $f \in N$. In the special case of biorthogonal bases spaces there is a particularly simple representation of continuous linear transformation [2; 17]. A transformation a is in $L(M, N)$ if and only if the matrix of a with respect to the basis $\{x_i\}$ is row and column finite.

Let $L(M)$ denote the ring of *all* linear transformations on M and $F(M)$ the ring of all finite-valued linear transformations in $L(M)$. It is known [4; 789] that any two-sided ideal I of $L(M)$ consists of all those linear transformations of rank less than some infinite cardinal $\aleph_I$ where $\aleph_I \leq \dim M$. From this it can be proven [6;18] that $L(M)/F(M)$ is primitive without one-sided minimal ideals, further that the quotient of any two two-sided ideals in $L(M)$ has this structure. In this paper we show that exactly the same results hold if M and N are biorthogonal bases spaces. In the final theorem we show that $L(M, N)$ and its ideals are not isomorphic to $L(M)$ and its ideals. In fact, we show that if $I_1 \supset I_2$ are two ideals in $L(M, N)$, then I_1/I_2 is not regular while it is known that the corresponding object for $L(M)$ is regular. (The symbols "$\supset$" and "$\subset$" are used in the strong sense, excluding equality.)

We would like to point out that Orenstein (unpublished) has shown that a closed subspace K of M has a closed complement such that the annihilators of K and this complement add up to all of N. We make no use of this fact in this work.

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2. **Notation and preliminary lemmas.** Let dim $M = \aleph$ where $\aleph$ is an infinite

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