ALGEBRAS WITH UNIQUE MINIMAL FAITHFUL REPRESENTATIONS

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Introduction. Let $\mathfrak{A}$ be a finite dimensional algebra with unit element over a field Φ. A faithful representation $\mathfrak{F}$ of $\mathfrak{A}$ is a minimal faithful representation if the deletion of any component of $\mathfrak{F}$ leaves a non-faithful representation of $\mathfrak{A}$. If $\mathfrak{A}$ has a unique minimal faithful representation then $\mathfrak{A}$ is called a UMFR algebra or an algebra of type UMFR. The generalized uniserial algebras form a subclass of the UMFR algebras. This paper gives characterizations of the various subclasses of the UMFR algebras which contain the generalized uniserial algebras as a subclass.

§1 contains the definitions and notations for the paper, while §2 contains the definitions of the sixteen subclasses to be studied. §3 gives proofs that at most twelve of these classes are distinct and that one of these "new" classes is the class of generalized uniserial algebras. §4 contains examples to show that these twelve classes are distinct. §5 gives characterizations of some of these classes in terms of conditions placed upon the socles of the primitive ideals of the algebras.

1. **Definitions and notations.** The word *ideal* will be used to mean either right or left ideal but not two-sided ideal. Thus, if it is stated that a condition holds for all ideals of a certain type, this will mean that the condition holds for all left ideals and all right ideals of that type. A *primitive* ideal is one which is generated by a primitive idempotent. A primitive left (or right) ideal is a *dominant* ideal if it is dual to some primitive right (or left) ideal. When referring to ideals of $\mathfrak{A}$ or to $\mathfrak{A}$-modules, the terms *homomorphic* and *isomorphic* will be used to mean homomorphic and isomorphic when considered as $\mathfrak{A}$-modules.

A primitive ideal $\mathfrak{J}$ is *subordinate* to an ideal $\mathfrak{J}'$ if there exists a subideal $\mathfrak{J}^*$ of $\mathfrak{J}'$ such that $\mathfrak{J}$ is isomorphic to $\mathfrak{J}^*$. A primitive ideal $\mathfrak{J}$ is *weakly subordinate* to a set of ideals $\{\mathfrak{J}_i\}_{i=1}^{n}$ if there exists a set of ideals $\{\mathfrak{J}_j'\}_{j=1}^{m}$ with each $\mathfrak{J}_j'$ a subideal of some $\mathfrak{J}_i$, such that $\mathfrak{J}$ is isomorphic to some submodule $\mathfrak{M}^*$ of the direct sum $\mathfrak{M} = \sum_{j=1}^{m} \mathfrak{J}_j'$ of the $\mathfrak{J}_j'$. Note that the $\mathfrak{A}$-module $\mathfrak{M}$ need not be isomorphic to a subideal of $\mathfrak{A}$ even though each $\mathfrak{J}_j'$ is a subideal of $\mathfrak{A}$.

A faithful representation $\mathfrak{F}$ of $\mathfrak{A}$ is *minimal* if the deletion of any component leaves a non-faithful representation of $\mathfrak{A}$. If there is only one such minimal faithful representation of $\mathfrak{A}$, then $\mathfrak{A}$ is called a *unique minimal faithful representation* algebra, written UMFR algebra. These algebras were first introduced by Thrall [3] and were called QF-3 algebras. He showed [3, Theorem 5] that an algebra $\mathfrak{A}$ is UMFR if and only if every primitive ideal $\mathfrak{J}$ of $\mathfrak{A}$ is weakly subordinate to a set $\{\mathfrak{J}_i\}_{i=1}^{n}$ of dominant ideals of $\mathfrak{A}$.

An algebra $\mathfrak{A}$ is *generalized uniserial* if and only if every primitive ideal of $\mathfrak{A}$

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