RETRACTS FROM NEIGHBORHOOD RETRACTS

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1. **Introduction.** Linda Falcao Foulis [4] answered in the affirmative Wojdyslawski's question [11] of whether the space of closed subsets of a CAR* (= retract of a Tychonoff cube) is a CAR*. We establish here that if X is a connected CANR* (= closed neighborhood retract of a Tychonoff cube), then $S(X)$ (= space of closed subsets of X) has the fixed point property, and that if X is metric, then $S(X)$ is a CAR*. We conjecture that the metric condition in the last statement can be dropped. Analogies between neighborhood retracts and the null-homotopic fixed point property suggest the question: is the fixed point property in $S(X)$ equivalent to the null-homotopic fixed point property in X?

2. **Retracts from neighborhood retracts.**

THEOREM 1. *Let X be a connected metric space. If X is a* CANR*, *then* $S(X)$ *is a* CAR*.

Proof. Since X is a retract of an open set U of a metrizable Tychonoff cube T, and since T is locally connected, U contains a closed, connected, locally connected set, P, which retracts onto X. It follows [9; 26] that X is locally connected. Therefore, [7, Theorem 8] X is an MCAR* and [8, Theorem 3] $S(X)$ is a CAR*.

The hypothesis that X be connected cannot be deleted from Theorem 1. It is easy to see that any finite space is a CANR*, but its space of closed subsets is not a CAR*.

That the converse of Theorem 1 does not hold is established by the following example. Let

$$A_{-2} = \{(x, x): \ 0 \le x \le 1\},$$
$$A_{-1} = \{(x, 0): \ 0 \le x \le 1\},$$
$$A_n = \{(2^{-n}, y): \ 0 \le y \le 2^{-n}\}, \qquad n = 0, 1, \cdots .$$

We define

$$X = \bigcup_{n=-2}^{\infty} \{A_n\}.$$

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