NOTE ON WEAK CUTPOINTS IN CLANS

By R. J. Koch

A clan is a compact connected Hausdorff topological semigroup with unit. It is a conjecture of the author and of A. D. Wallace that if S is a clan which is not a group, then the unit cannot be a weak cutpoint (see definitions below), and it is a further conjecture that the unit cannot lie in a non-trivial c-set. It is the object of this note to establish the equivalence of these conjectures, and to answer them affirmatively in case S is either one-dimensional or homogeneous. In the one-dimensional case we show further that the unit of S does not cut any subcontinuum of S.

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An element a of a continuum X is a *weak cutpoint* if there exist $x, y \in X \setminus a$ such that any subcontinuum of X containing x and y also contains a. For $p \in X$, let $T_p = \{x \in X \mid \text{if } M \text{ is a continuum with } x \in M^0, \text{ then } p \in M\}$ (where the interior is taken relative to X). In the terminology of Jones [3], T_p is the set of all points at which S is non-aposyndetic with respect to p, and T_p is known to be a subcontinuum. These subcontinua have been investigated in clans in [5], and will be dealt with further here.

A subset C of X is a c-set if for each subcontinuum M of X with $M \cap C \neq \square$, either $C \subset M$ or $M \subset C$. A c-set is known to be connected, and if $C \neq X$, then C contains no inner point. C-sets in clans have been investigated by Wallace [10]

We follow the terminology of [9]. In particular, K denotes the minimal (closed) ideal of a compact mob, E denotes the set of all idempotents, and for $e \in E$, H_e is the maximal subgroup containing e.

Definition. Let S be a clan (with unit u) which is not a group. Define $W_u = \{x \mid \text{if } M \text{ is a continuum with } x \in M \text{ and } M \not\subset H_u, \text{ then } u \in M\}$.

Lemma 1. *Let S be a clan (with unit u) which is not a group; then $T_u = W_u \subset H_u$.*

Proof. Suppose $x \in W_u$, and let M be a continuum with $x \in M^0$. Let $J = S \setminus H_u$; then J is an open dense ideal [4], so $M^0 \cap J \neq \square$. Hence $u \in M$, so $x \in T_u$. Now take $x \in T_u$ and let M be a continuum with $M \not\subset H_u$ and $x \in M$. Choose $b \in M \setminus H_u$ and note that $M \cup SbS$ is a continuum containing K. Suppose $u \in S \setminus M$; then $u \in S \setminus u(M \cup SbS)$, so there is an open set V about u with $u \in S \setminus V^*(M \cup SbS)$. Now $V^*(M \cup SbS) = \cup \{y(M \cup SbS) \mid y \in V^*\}$, and each set $y(M \cup SbS)$ is a continuum meeting K, so it follows that $V^*(M \cup$

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