AN IMBEDDING THEOREM

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1. **Introduction.** Although 2-dimensional (separable metric) generalized closed manifolds are topological manifolds, 3-dimensional generalized closed manifolds need not be (see Wilder [7; 245]). Certain additional conditions have been suggested which might insure that such a manifold is a topological manifold. Wilder [8] suggested the condition of imbeddability in E^4, and Griffiths [5] has suggested some local homotopy conditions. It is shown by Curtis-Wilder [3] that neither of these conditions alone will suffice. In this note we show that the combination of the two will not suffice by showing that a certain decomposition space defined by Bing [1], and shown in [3] to satisfy Griffiths' conditions, is imbeddable in E^4.

2. An imbedding theorem.

Definition. A homotopy $\phi: X \times I \rightarrow Y$ is called a *pseudo-isotopy* if ϕ_t is a homeomorphism for $0 \leq t < 1$.

The following theorem is a consequence of a result of Floyd-Fort [4] (see the remark by Bing in [2; 284]).

Theorem 1. *Let $\phi: E^2 \rightarrow E^2$ be monotone and onto. Then there exists a pseudo-isotopy $\Phi: E^2 \times I \rightarrow E^2$ such that Φ_0 is the identity and $\Phi_1 = \phi$.*

Corollary 1. *Let $\phi: E^2 \rightarrow E^2$ be monotone and onto. Let y_0 be a point in a metric space Y. Then there exists a pseudo-isotopy $\Phi: E^2 \times Y \times I \rightarrow E^2 \times Y$ such that:*

(1) *Φ_0 is the identity*
(2) *Each Φ_t maps each $E^2 \times y$ onto itself*
(3) $\Phi_1 \mid E^2 \times y_0 = \phi$.

Proof. Let r be a metric in Y, and let $F: E^2 \times I \rightarrow E^2$ be a pseudo-isotopy furnished by Theorem 1. For $y \in Y$ such that $r(y_0, y) > 1$, we define $\Phi(q, y, t) = (q, y)$. For $y \in Y$ such that $r(y_0, y) \leq 1$ we define

$$\Phi(q, y, t) = (F[q, t(1 - r(y_0, y))], y).$$

Φ_0 is the identity since F_0 is the identity and (2) follows from the corresponding property of F. (3) follows since

$$\Phi_1(q, y_0) = (F[q, 1], y_0) \quad \text{and} \quad F_1 = \phi.$$

Theorem 2. *Let E^2 be a linear subspace of E^k. Suppose $\pi: E^k \rightarrow M$ is mono-*

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