A BOUNDARY VALUE PROBLEM OF PARTIAL DIFFERENTIAL EQUATIONS OF PARABOLIC TYPE

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Introduction. The theory of linear parabolic differential equations has been discussed by many scholars in different approaches. Recently, the author has shown the existence and uniqueness of the solution of homogeneous parabolic differential equations of second order in a compact domain associated with a boundary condition of homogeneous mixed Dirichlet-Neumann type. (See [10, II].) In the present paper, we shall extend the previous result to the case of inhomogeneous linear parabolic equations and inhomogeneous boundary conditions. Some of the assumptions on the coefficients made in our previous paper [10,II] are omitted here.

We shall construct the fundamental solution of the parabolic equation (the construction being the same as Feller [5] and Dressel [4] used except for statements concerning the boundary condition), and give an explicit formula to express the solution of the parabolic equation with given initial condition and boundary condition.

1. **Fundamental notions and main results.** Let $[s_0\ ,\ t_0]$ be a bounded closed interval, M be an m-dimensional orientable manifold of class C^∞, and D be a domain in M whose closure $\bar{D}$ is compact and whose boundary $B = \bar{D} - D$ consists of a finite number of hypersurfaces of $m - 1$ dimensions and of class-C^3.

We understand the partial derivatives at $\xi \in B$ of a function $f(x)$ defined on $\bar{D}$ as follows: $\partial f(\xi)/\partial x^i = \alpha_i$ $(\xi \in B)$, $i = 1, \cdots, m$, mean that

$$f(x) = f(\xi) + \alpha_i(x^i - \xi^i) + o(\sum_i \mid x^i - \xi^i \mid)$$

for any $x \in U(\xi) \cap \bar{D}$, where $U(\xi)$ is a coordinate neighborhood of ξ. (We omit the summation sign $\sum$ when and only when the same indices appear as both subscript and superscript.)

We consider the parabolic differential operator L on $[s_0\ ,\ t_0] \times \bar{D}$:

$$(L) \qquad L = L_{t,x} = A_{t,x} - \frac{\partial}{\partial t}$$

where

$$(A) \qquad A = A_{t,x} = a^{ij}(t, x) \frac{\partial^2}{\partial x^i\, \partial x^j} + b^i(t, x) \frac{\partial}{\partial x^i} + c(t, x);$$

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