IDEALS IN MULTIPLICATIVE SYSTEMS OF CONTINUOUS FUNCTIONS: A CORRECTION

BY PAUL CIVIN AND BERTRAM YOOD

The proof of Lemma 5.2 of [1] is in error. This necessitates some minor changes in that Lemma and the subsequent material of §5. To that end we say that a semi-group B in C is an *exterior semi-group* if every $g \in B$ is the limit of a sequence $\{g_n\}$ in B^c , sup (g_n) compact and int $Z(g_n) \supset Z(g)$. If B is dense in C it is automatically an exterior semi-group.

5.2. LEMMA. Let I be a T-ideal in an exterior semi-group B. Then I = kh(I).

Let $g \in kh(I)$, $g = \lim g_n$ as per assumption. For any integer n, $\sup (g_n) \subset X - h(I) = X - h(I_0) = \bigcup (X - Z(f))$, $f \in I_0$ (see [1; 326] for notation). Thus there exist f_1, \dots, f_n in I_0 such that $\sup (g_n) \subset \bigcup_{1}^{n} (X - A(f_i))$. Let $\{e_m\} \in I_0$, $e_m \cup w \in B^c$, $wf_i = f_i$, $j = 1, \dots, r$. Since w(t) = 1, $t \in \sup (g_n)$, $\lim_{n \to \infty} e_m g_n = g_n$. Now $g_n = \lim h_{n,s}$, $h_{n,s} \in B$. Thus $e_m h_{n,m} \to wg_n = g_n$. Therefore $g_n \in I_0^c$ (closure in C) whence so is g. Since $g \in B$, $g \in I$.

5.3 THEOREM. A semi-group B in C is normal, general Urysohn and exterior if and only if there is a one-to-one correspondence between the T-ideals I of B and the closed sets F of X such that if $I \leftrightarrow F$ then I = k(F).

Assume the correspondence. By [1; 330] it suffices to show B is exterior. Let $g \in B$, F = Z(g). $g \in k(F)$ which is a T-ideal. Thus $g = \lim g_n$, $g_n \in [k(F)]_0$. Thus sup (g_n) is compact. Fix n. There exists a sequence $\{e_m\} \in [k(F)]_0$, $e_m \to w \in B^c$ such that $wg_n = g_n$. Now int $Z(g_n) \supset Z(w)$. But $Z(w) \supset F$. Thus B is exterior.

For the converse the argument remains as before with use of Lemma 5.2 as given above.

The first sentence in Theorem 5.4 should read as follows. Let B be a general Urysohn and exterior semi-group. For Theorem 5.5 and Corollary 5.6 it should be assumed that the representation algebra $\pi(A)$ is exterior. The latter results still apply to the group algebra of a locally compact Abelian group since the representation algebras are dense and hence exterior.

References

1. PAUL CIVIN AND BERTRAM YOOD, Ideals in multiplicative systems of semi-groups, this Journal, vol. 23(1956), pp. 325-334.

UNIVERSITY OF OREGON

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