SEPARATION PROPERTIES AND n-INDECOMPOSABLE CONTINUA

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A continuum M is said to be *n-indecomposable* if (1) M is the union of n continua such that no one of them is a subset of the union of the others and (2) M is not the union of $n + 1$ such continua. In some previous papers [3, 4, 5, 11], such a continuum has been said to be *indecomposable under index n*. Swingle [11, Theorem 2] has shown that every n-indecomposable continuum is the union of n indecomposable continua such that no one of them is a subset of the union of the others, and the author [4, Theorem 2] has shown that these n indecomposable continua are unique in this respect. In accordance with the above definition, every indecomposable continuum is 1-indecomposable.

A subset H of a continuum M is said to *cut M between two subsets K and L* of M if K and L lie in $M - H$, and H intersects every subcontinuum of M which intersects both K and L.

A subset H of a continuum M is said to *cut M* if H cuts M between some two points.

A subset H of a continuum M is said to *separate M* if $M - H$ is not connected.

A continuum M is said to be *separated* (*cut*) *by the pair of continua H* and K if $H \cup K$ separates (cuts) M.

A subset H of a point set M is said to be an *open subset* of M if H is open relative to M.

THEOREM 1. *If, for some positive integer n, the compact metric continuum M is n-indecomposable, then every proper subcontinuum of M cuts M.*

Proof. Let G be a collection consisting of n indecomposable continua filling up M such that no one of them is a subset of the union of the others, and let H be any proper subcontinuum of M. It can easily be seen that some continuum K of G intersects H but does not lie in H. It follows from [3, Theorem 1] that some two composants of K lie in $M - \cup (G - K)$ and do not intersect H. (The collection of all elements of G different from X is denoted by $G - X$.) Let x be a point of one of these two composants and y a point of the other, and let L be a subcontinuum of M containing $x \cup y$. It readily follows that $L \supset K$ and, consequently, that L intersects H. Hence H cuts M.

THEOREM 2. *If M is a bounded plane continuum such that no pair of subcontinua of M separates M and no subcontinuum of M cuts M between two open subsets of M, then there is a positive integer n less than five such that M is n-indecomposable.*

Proof. Suppose that M is the union of five continua $M_1, M_2, \cdots, M_5$ such that no one of them is a subset of the union of the others. There exist

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