WEIGHTED QUADRATIC PARTITIONS OVER $GF[q, x]$

By L. Carlitz

1. **Introduction.** Let $q = p^n$, $p > 2$, and let $GF[q, x]$ denote the ring of polynomials in x with coefficients in $GF(q)$. Let $\Phi = GF\{q, x\}$ denote the power series field consisting of the quantities

$$\sum_{-\infty}^{m} c_i x^i \qquad (c_i \in GF(q)) \tag{1.1}$$

We consider the following problem. Let $r \geq 1$, $\xi_1, \cdots, \xi_r$ arbitrary numbers of Φ, $a_1, \cdots, a_r$ non-zero numbers of $GF(q)$. Then if k is a fixed integer ≥ 1 and M a polynomial of $GF[q, x]$ of degree $\leq 2k$, define the weighted sum

$$\sum e(2U_1\xi_1 + \cdots + 2U_r\xi_r), \tag{1.2}$$

where the summation is over all $U_i \in GF[q, x]$ satisfying

$$a_1U_1^2 + \cdots + a_rU_r^2 = M \tag{1.3}$$

as well as certain auxiliary conditions. To define $e(\xi)$, let ξ be the number (1.1); then

$$e(\xi) = e^{2\pi i t(c_{-1})/p} \qquad t(c) = c + c^p + \cdots + c^{p^{n-1}}. \tag{1.4}$$

As for the auxiliary conditions, let $0 \leq l \leq r$, then it is assumed in (1.3) that $U_1, \cdots, U_l$ are primary of degree k while $U_{l+1}, \cdots, U_r$ are arbitrary of degree $< k$. Cohen [5], [6], [7] has shown that in the problem of the number of solutions of (1.3) there is a fundamental distinction between the case $l \geq 1$ and the case $l = 0$. The sum (1.2) reduces to the number of solutions of (1.3) when $\xi_1 = \cdots = \xi_r = 0$, so that one may expect that this difference between the two cases will appear for arbitrary ξ's.

In an earlier paper [4] the writer has discussed the sum

$$\sum e_0(2b_1x_1 + \cdots + 2b_rx_r), \tag{1.5}$$

where the sum is over all $x_i \in GF(q)$ such that

$$a_1x_1^2 + \cdots + a_rx_r^2 = a \qquad (a_i, a \in GF(q)), \tag{1.6}$$

and

$$e_0(a) = e^{2\pi i t(a)/p}, \qquad t(a) = a + a^p + \cdots + a^{p^{n-1}}.$$

It was shown that (1.5) can either be evaluated explicitly, or at any rate, expressed in terms of a Kloosterman sum over $GF(q)$. The sum (1.5) is evidently a special case of (1.2). In the present paper we show how (1.2) can be evaluated.

Received May 9, 1956.