NOTE ON AN INEQUALITY OF PRAWITZ

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1. In the present note it is shown how an inequality of Prawitz gives rise to a well-known class of schlicht functions, as stated in Theorem 1. As an application of Theorem 1, it is shown in §2 how the derivation of another class of schlicht functions can be simplified.

Let (S) denote the class of functions $w = f(z) = z + a_2z^2 + a_3z^3 + \cdots$ which are regular and schlicht for $|z| < 1$. Form

$$(1.1) \qquad F(z) = \left(\frac{f(z)}{z}\right)^{-\alpha/2} = 1 + \sum_1^\infty b_n z^n,$$

then by Prawitz [3], it follows that

$$(1.2) \qquad \sum_1^\infty \frac{1}{\alpha}(2n - \alpha) \mid b_n \mid^2 \leq 1, \qquad \text{for all real } \alpha.$$

In particular, for $\alpha = 2$, we obtain

$$(1.3) \qquad \mid b_2 \mid^2 + 2 \mid b_3 \mid^2 + 3 \mid b_4 \mid^2 + \cdots \leq 1.$$

Using the above notation, we prove

Theorem 1. *Let* $f(z) \subset (S)$. *Let* $\sum_1^N (2n - \alpha) \mid b_n \mid^2/\alpha = 1$, $\mid b_N \mid = 1$. *Then*

$$(1.4) \qquad f(z) = \frac{z}{\prod_{j=1}^{N} (ze^{i\theta_j} - 1)^{2/N}}.$$

To prove Theorem 1, set $\sum_1^N (2n - \alpha) \mid b_n \mid^2/\alpha = 1$ and solve for $\alpha/2$,

$$(1.5) \qquad \frac{\alpha}{2} = \frac{\sum_1^N kB_k^2}{1 + \sum_1^N B_k^2}; \qquad B_k = \mid b_k \mid$$

so that $f(z)$ becomes

$$f(z) = \frac{z}{(P(z))^{2/\alpha}} = \frac{z}{\left(1 + \sum_1^N b_k z^k\right)^{2/\alpha}}$$

where $2/\alpha$ is given by (1.5). Since $f(z)$ is regular for $|z| < 1$, and the product of the zeros of $P(z) = \mid b_N \mid = 1$, it follows that all the zeroes of $P(z)$ lie on the

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