EMBEDDING OF AW^* ALGEBRAS

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1. **Introduction.** In [5], I. Kaplansky axiomatizes a class of C^* algebras which he calls AW^* algebras. For these, much of the Murray-von Neumann structure theory for rings of operators can be carried through. Every ring of operators, regarded as an abstract C^* algebra, is an AW^* algebra. The converse question, whether every AW^* algebra has some faithful representation as a ring of operators, has a negative answer on a relatively superficial level: a commutative AW^* algebra can be so represented if and only if its spectrum is "hyperstonnien" [2, Theorem 2]; and, indeed, there exist commutative AW^* algebras with non-hyperstonnien spectra, a familiar example being the bounded Baire functions on the real line, where two functions are identified when they agree except on a set of the first category. Theorem 1 of the present paper states that a finite AW^* algebra with a separating set of states which are completely additive on projections can be represented as a ring of operators. (One would suspect that the theorem is true without the assumption of finiteness). Theorem 2 states, for arbitrary AW^* algebras which possess separating sets of states which are completely additive on projections, that they can at any rate be AW^* embedded as operators on Hilbert space. It should be mentioned here that R. V. Kadison shows, in a paper to appear in the *Annals of Mathematics*, that if one makes the stronger assumptions on a C^* algebra that every descending hypersequence $\{A_i\}$ of nonnegative operators have a g.l.b. A, and that there exist a separating family of states ω such that $\omega(A_i) \uparrow \omega(A)$ for any such hypersequence, then the algebra is faithfully representable as a ring of operators.

2. **Definitions and preliminaries.** For information about AW^* algebras, AW^* embedding, and equivalence of projections, see [5]; for information about rings of operators, see [6].

Lemma 1. *If the AW^* algebra $\mathbf{A}$ is AW^* embedded in the ring of all bounded operators on the Hilbert space $\mathfrak{H}$, then for any self-adjoint element $A = \int rdE(r)$ of $\mathbf{A}$, the spectral projections $E(r)$ are in $\mathbf{A}$.*

Proof. Let $\mathbf{C}$ be any commutative AW^* algebra. Being a C^* algebra, $\mathbf{C}$ is isomorphic via a canonical isomorphism ϕ to the C^* algebra $\mathbf{C}(\Gamma)$ of all continuous functions on its spectrum Γ. Since $\mathbf{C}$ is AW^*, Γ is compact and extremely disconnected [5]. If A is self-adjoint in $\mathbf{C}$, let $\Gamma(r)$ be the interior of $\{\gamma \varepsilon \Gamma \mid \phi(A)\ (\gamma) \leq r\}$. $\Gamma(r)$ is both closed and open, and its characteristic function f_r is therefore continuous. Let $F(r)$ be $\phi^{-1}(f_r)$. Then $F(r)$ is a monotone nondecreasing projection-valued function of r with supremum I and

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