A GEOMETRIC CONSTRUCTION OF L-SPACES

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1. **Introduction.** In recent years considerable attention has been given to investigations of topological properties of normed linear spaces. Of special interest has been the problem of determining when two Banach spaces are homeomorphic. One of the early results in this direction, for example, is due to S. Mazur [8] showing that L^p $(1 < p < \infty)$ is homeomorphic to L^1. The main result of our paper, Theorem 4.3, is somewhat related to the general problem since it gives necessary and sufficient conditions that a Banach space X can be renormed to an L-space homeomorphic to X under the identity transformation. The geometric construction referred to in the title of this paper, and called the L-construction hereafter, is defined in §3 and discussed in §4. The chief justification for the L-construction is its use in proving Theorem 4.3 although, as will be seen in §4, other results of some interest can be derived by applying the L-construction in appropriate situations. Some necessary background material is given in §2 while §5, somewhat independent of the other sections, demonstrates that the positive cone in any infinite dimensional L-space has a void interior. It should be mentioned that this last result seems to be known in some quarters but, as far as we know, has not previously been demonstrated in the literature.

2. **Preliminaries.** We will consider real linear spaces X with vector elements $x, y, \cdots$ and real scalars $a, b, \cdots$. The following notation and terminology will be employed. The additive identity of a linear space will be denoted by Θ to distinguish it from the real number 0. Set theoretic sum, difference and product are indicated by $\cup$, $\setminus$ and $\cap$ respectively; $+$, $-$ and $\cdot$ (juxtaposition) being reserved for the linear operations. If A is a subset of X, x is an element of X and a is a real scalar then $x + A = \{x + y : y \in A\}$ and $aA = \{ax : x \in A\}$. The *convex hull* of A is the smallest convex set containing A, consists of all finite convex combinations of elements in A and is denoted by $\langle A \rangle$.

A normed linear space X with norm $\| x \|$ and partial ordering $x \geq y$ under which X is a linear lattice (c.f. G. Birkhoff [2]) is called a *normed linear lattice* if also

$$(1) \qquad x_n \geq y_n, \quad \| x_n - x \| \to 0, \quad \| y_n - y \| \to 0 \quad \text{imply} \quad x \geq y.$$

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