SIMILARITY PRINCIPLE WITH BOUNDARY CONDITIONS FOR PSEUDO-ANALYTIC FUNCTIONS

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1. **Introduction.** A basic result in the theory of pseudo-analytic functions is the similarity principle proved by Bers [3]. In essence, the similarity principle states that with every pseudo-analytic function w can be associated an analytic function f (and vice versa) such that the ratio w/f is bounded, bounded away from zero and continuous and such that w/f can be made to satisfy certain boundary conditions. We give here an extension of the similarity principle; the boundary conditions are now imposed on several boundary curves. Applications to elliptic partial differential equations will then be given.

Throughout this paper, we assume D to be a domain bounded by $n + 1$ smooth curves Γ_j, $j = 0, 1, \cdots, n$. A complex-valued function f is said to be of class H on D if f satisfies a uniform Hölder condition on every compact subset of D. Let a and b be two functions of class H on D. (In what follows the condition that a and b be of class H can be weakened to a, b measurable and of class L^p, $p > 2$; cf. [5] and the forthcoming thesis of I. Polonsky.) A complex-valued function w of class C^1 on D is called $[a, b]$ pseudo-analytic (of the first kind) if

$$w_{\bar{z}} = aw + b\bar{w}.$$

In this expression as well as in what follows the formal differential operators $\partial/\partial z$ and $\partial/\partial \bar{z}$ are used where $w_z = \frac{1}{2}(w_x - iw_y)$ and $w_{\bar{z}} = \frac{1}{2}(w_x + iw_y)$. A pseudo-analytis function with poles on D will be called pseudo-meromorphic.

Let ρ be a measurable function on D and set

$$I_D(\rho \,||\, z) = -\frac{1}{\pi} \int_D \frac{\rho(\zeta)}{\zeta - z}\, d\mu(\zeta); \qquad z \in \overline{D}$$

where μ denotes two-dimensional Lebesgue measure. A non-negative function K of z is said to be *admissible* (cf. [3]) on D if for every measurable function ρ such that $|\rho(z)| \leq K(z)$, there exist positive constants M and ϵ, $0 < \epsilon < 1$, depending only on D and K such that $|I_D(\rho \,||\, z)| \leq M(1 + |z|)^{-\epsilon}$; $z \in \overline{D}$ and

$$|I_D(\rho \,||\, z_1) - I_D(\rho \,||\, z_2)| \leq M\, |z_1 - z_2|^{\epsilon}; \qquad z_1, z_2 \in \overline{D}.$$

We begin with a known lemma [3; 71].

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