RESTRICTED SUMMATION AND LOCALIZATION OF DOUBLE TRIGONOMETRIC SERIES

By Leonard D. Berkovitz and Richard P. Gosselin

In the work of Riemann, Rajchman, and Zygmund, an elegant theory of localization for trigonometric series of a single variable has been achieved. (See [7] for complete references.) In considering the corresponding problem for two variables, one is faced with the problem, among other things, of choosing a method of summability. Lepecki [3] and the second author [2] have considered the localization problem for series summed by Pringsheim methods and have obtained results which involve cross-shaped neighborhoods, while the first author [1] has obtained a localization theory which requires only the ordinary type of neighborhood by using Bochner's circular summation. However, difficulties arise in certain natural extensions of this work. For example, V. Shapiro [5] has proved a localization theory using square summation which fails for higher orders.

It is our aim in this paper to present a theory of localization for double trigonometric series involving the usual type of neighborhood and using double index summation methods. Our choice of method is that of restricted Riesz summability with kernel $(1 - t^2)^\delta$.

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1. **Notations and definitions.** Following C. N. Moore [4], we say that a double sequence S_{MN} converges to a limit L restrictedly, or in the restricted sense, if for any fixed positive constants A and B there correspond to every $\epsilon > 0$ positive integers N_ϵ and M_ϵ, with the property that $| S_{MN} - L | < \epsilon$ for all $M > M_0$ and $N > N_0$ whose ratio satisfies the inequality $A \leq M/N \leq B$. A double series $\sum_{m,n=-\infty}^{+\infty} a_{mn}$ is said to converge to L restrictedly if the sequence of partial sums S_{MN} does, where $S_{MN} = \sum_{m,n=-M,-N}^{M,N} a_{mn}$. Henceforth, when considering double series whose limits of summation are $\pm\infty$, we shall not bother to put down the limits in cases where no confusion can result. Thus we shall merely write $\sum a_{mn}$ instead of $\sum_{m,n=-\infty}^{+\infty} a_{mn}$.

Consider a sequence a_{mn}, where $-\infty < m < +\infty$ and $-\infty < n < +\infty$. By the statement $a_{mn} = o\,((\,|\,m\,|\, + 1)^\gamma\,(\,|\,n\,|\, + 1)^\delta)$ we mean the following: first that there exists a constant $K > 0$ such that $|\,a_{mn}\,| \leq K((\,|\,m\,|\, + 1)^\gamma\,(\,|\,n\,|\, + 1)^\delta)$ for all m and n, and secondly that given an $\epsilon > 0$ there exist positive integers m_0 and n_0 such that $|\,a_{mn}\,| \leq \epsilon\,((\,|\,m\,|\, + 1)^\gamma\,(\,|\,n\,|\, + 1)^\delta)$ whenever $n > n_0$ and $m > m_0$.

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