A PROPERTY OF COMPACT ABSOLUTE NEIGHBORHOOD RETRACTS

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1. **Introduction.** It is not difficult to construct examples of topological spaces X which admit of fixed point free (fpf) mappings (= continuous mappings) $f: X \to X$ but do not admit of fpf mappings f which are *onto*, i.e. mappings f such that $f(X) = X$. For example, let X denote the half-line $0 \leq x < \infty$ and $Y = C \cup S$, where C is the unit circle in the plane and S a spiral interior to C and asymptotic to C. Then X and Y, with their natural topologies, have the desired property. One is, therefore, led to the problem of finding appropriate conditions on topological spaces to insure the existence of *onto* fpf mappings provided any fpf maps exist at all. One simple condition is that the space be a multicoherent Peano continuum. For, following Kuratowski [3], one can easily construct the desired *onto* fpf mappings directly by slightly modifying the proof in [3]. The purpose of this paper is to prove the following theorems which are immediate corollaries of a more general theorem given in §3.

Theorem: *If X is a compact connected ANR (absolute neighborhood retract) and $f: X \to X$ a fpf mapping, then there exists a mapping $g: X \to X$ such that $g(X) = X$ and $f \sim g$ (f homotopic to g) with homotopy H such that H_t is fpf for all t, $0 \leq t \leq 1$.*

Theorem: *If X is a cyclic (no cut points) compact connected ANR and $f: X \to X$ is* any *mapping, then there exists a mapping $g: X \to X$ such that $g(X) = X$ and $f \sim g$ with homotopy H such that the fixed points of H_t are precisely those of f for all t, $0 \leq t \leq 1$.*

2. **Definitions and preliminaries.** All topological spaces hereafter considered will be non-degenerate and metric. Also, we shall use ANR in the sense of ANR (compact metric) even though the classical results on ANR's required here remain true for ANR (fully normal), (see [1]). The need for results outside the ANR theory requires the use of a Peano continuum [4] and hence there is no point in employing a more general class of ANR's. Furthermore, even though we will not assume an ANR connected, we will assume that its components are non-degenerate. The reason for this is made clear in the sequel.

The following lemma is well-known. A simple direct proof may be found in Yajima [5].

Lemma 1. *If X is an ANR, then for each $x \varepsilon X$ and open set U containing x, there exists an open set $V \subseteq U$ containing x such that for any pair (Y, B) (Y metric, B a closed subset of Y) of spaces, any mapping $f: B \to V$ has an extension to Y relative to U.*

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