CLASS FORMATIONS

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Introduction. Local class field theory as developed by Chevalley [2] and Nakayama [14] has been reconstructed by Hochschild [5] using the cohomology theory of groups. Nakayama and Hochschild [6], [15] showed that global class field theory can be formulated in a similar way. Finally, Artin and Tate [1] formulated the theory of class formation by which formal parts of global and local class field theory can be treated simultaneously. On the other hand, we have also (i) the theory of Abelian extensions over an infinite algebraic number field (see Moriya [12], Kawada [8]) and over an infinite complete field with respect to a valuation (see Moriya [11], Moriya and Schilling [13], Schilling [16]); (ii) the theory of Kummer extensions over a field which contains all the roots of unity (see Deuring [4], Kawada [9]); (iii) the theory of unramified Abelian extensions over an algebraic function field of one variable with complex constant field (see Weyl [19], Igusa [7]); and (iv) the theory of Abelian p-extensions over a field of characteristic p (see Witt [20] and Kawada [9]).

The purpose of this paper is to consider the relation between the theory of class formation and these theories of Abelian extensions. Thus we can unify, to some extent, various theories of Abelian extensions by means of class formation. In §1 we shall give necessary facts about class formations. Then in §2, §3, §4 and §5 we shall consider the relation between the theory of class formation and the above mentioned theories of Abelian extensions (i), (ii), (iii) and (iv) successively.

1. **Class formation.** We give here some necessary definitions and theorems concerning class formation which are more or less well known (see, in particular, Artin and Tate [1]). Let k_0 be a given ground field, and Ω be a fixed infinite separable normal algebraic extension of k_0 (which is not necessarily algebraically closed). Let $\mathfrak{K}$ be the set of all finite extensions of k_0 in Ω:

$$\mathfrak{K} = \{K; k_0 \subset K \subset \Omega, \qquad [K; k_0] < \infty\}.$$

DEFINITION. Let an additive group $E(K)$ be attached to every $K \in \mathfrak{K}$. If the following properties F1-F4 are satisfied, we call $\{E(K); K \in \mathfrak{K}\}$ a *formation*:

F1. If $k \subset K$ an isomorphism $\varphi_{k/K}$ of $E(k)$ into $E(K)$ is defined.

F2. For $k \subset l \subset K$ the relation $\varphi_{l/K} \circ \varphi_{k/l} = \varphi_{k/K}$ holds.

F3. Let K/k be normal and $G = G(K/k)$ be its Galois group, then G acts on $E(K)$ such that $\varphi_{k/K}(E(k)) = E(K)^G$ holds. Here we mean, in general, $A^G = \{a; a \in A, \sigma a = a \text{ for all } \sigma \in G\}$ for a G-module A.

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