# ORBIT SPACES OF FINITE TRANSFORMATION GROUPS. II

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In continuing our work of a previous paper [6], we shall prove theorems of the following type. Let $X$ be a finite-dimensional compact metric space, let $G$ be a finite transformation group on $X$, and let $Y$ be an orbit space for $(X, G)$. If $X$ is either (i) homologically trivial over $K$ where $K$ is a compact abelian group, or (ii) an absolute neighborhood retract, or (iii) an absolute retract, so also is $Y$. For references to previous work on problems of a related nature, see our first paper [6], which also contains sections 1 and 2.

3. **Covering theorems.** We assume throughout this section that *$X$ is a compact Hausdorff space, that $G$ is a finite transformation group on $X$, that $Y$ is an orbit space for $(X, G)$, and that $f{:}X \to Y$ is an orbit map.* If $H \subset G$ and $A \subset X$ then $HA$ or $H(A)$ will denote the set $[gx \mid g \varepsilon H,\ x \varepsilon A]$. A subset $A$ of $X$ is *invariant under* $G$ in case $GA = A$. A collection $U$ of subsets of $X$ is *invariant under* $G$ in case $u \varepsilon U$ and $g \varepsilon G$ implies $gu \varepsilon U$. We leave the proof of the following to the reader.

(3.1) *Suppose $U$ is an open covering of $X$. Let $A$ be a closed invariant subset of $X$ such that if $x \varepsilon A$ there exists a neighborhood $v$ of $x$ with $f$ one-to-one on $A \cap v$. There exists a finite invariant open covering $V$ of $A$ by sets open in $X$ which refines $U$ and is such that if $u$, $v$, $gu$ is a triple in $V$ with $\bar{u} \cap \bar{v} \neq \phi$ and $\bar{v} \cap g\bar{u} \neq \phi$, then $u = gu$.*

The central theorem of this section is the following.

(3.2) Theorem. *If $U$ is an open covering of $X$, there exists a finite open covering $V$ of $X$ which refines $U$ and is such that if $u, v \varepsilon V$, $g \varepsilon G$, and $u \cap v \neq \phi$, $v \cap gu \neq \phi$, then either $u = gu$ or $v = gv$.*

*Proof.* Let $n$ denote the order of $G$. Let $F(i)$, $0 \leq i \leq n$, be the set of all $x \varepsilon X$ such that $gx = x$ for at least $n - i$ distinct elements of $G$. Then $F(0) \subset F(1) \subset \cdots \subset F(n) = X$ in an increasing sequence of closed sets. By induction on $i$, we shall prove that there exists a finite covering $V_i$ of $F(i)$ consisting of sets open in $X$, and possessing the following properties:

(a) $V_i$ is invariant and refines $U$;

(b) if $u, v \varepsilon V_i$, $g \varepsilon G$, and $\bar{u} \cap \bar{v} \neq \phi$, $\bar{v} \cap g\bar{u} \neq \phi$, then either $u = gu$ or $v = gv$.

The theorem will follow with $V = V_n$.

The existence of $V_0$ is a consequence of (3.1). Suppose $V_{i-1}$ has been obtained, $i \geq 1$. For each $g \varepsilon G$, let $F_g = [x \mid x \varepsilon X,\ gx = x]$. For $h \varepsilon G$ we have $hF_g = F_{hgh^{-1}}$. As a consequence, $hF(i) = F(i)$, and $F(i)$ is invariant. Define $N_{i-1}$

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