THE COEFFICIENTS OF MEROMORPHIC FUNCTIONS

By Toshio Umezawa

1. **Introduction.** Recently A. W. Goodman [2] has conjectured the following: Let

$$(1.1) \qquad f(z) = z^q + \sum_{n=q+1}^{\infty} b_n z^n$$

be regular and p-valent in $|z| < 1$, and let $f(z)$ have s zeros, $\beta_1, \beta_2, \cdots, \beta_s$, such that $0 < |\beta_j| < 1, j = 1, 2, \cdots, s$. Finally let the non-negative integer t be defined by

$$(1.2) \qquad q + s + t = p \geq 1$$

and let

$$(1.3) \qquad \begin{aligned} F_p(z) &= \frac{z^q}{(1-z)^{2q+2s}} \left(\frac{1+z}{1-z}\right)^{2t} \prod_{j=1}^{s} \left(1 + \frac{z}{|\beta_j|}\right)(1 + z|\beta_j|) \\ &= z^q + \sum_{n=q+1}^{\infty} B_n z^n \end{aligned}$$

then

$$(1.4) \qquad |b_n| \leq B_n \qquad (n = q+1, q+2, \cdots).$$

The inequality (1.4) has been proved to be valid in the special cases that $t = 0$ and $f(z)$ is starlike of order p in the direction of the diametral line [7].

In Theorem 2 of this paper we shall generalize the above results to the case of meromorphic functions, proving that the magnitude of its Laurent series coefficients depends on the location of its zeros and poles.

On the other hand M. S. Robertson [5] has recently obtained the following

Theorem A. *Let*

$$(1.5) \qquad w = f(z) = \sum_{-\infty}^{\infty} a_n z^n$$

be regular and single-valued for $0 \leq \rho < |z| < 1$. *On each circle* $|z| = r$, $\rho < r < 1$, *let the imaginary part of* $f(z)$ *change sign* $2p$ *times, where* p *is a positive integer independent of* r. *Then, for* $n > p$, *the following inequalities hold and are sharp in all the variables* $|a_k - \bar{a}_{-k}|$ $(k = 0, 1, \cdots, p)$

$$(1.6) \qquad |a_n - \bar{a}_{-n}| \leq \sum_{k=0}^{p} \Delta(p, k, n) \, |a_k - \bar{a}_{-k}|,$$

Received March 27, 1953.