THE SUMMABILITY FACTORS OF INFINITE SERIES

By T. Pati

1.1. Definitions. Let $\sum a_n$ be a given infinite series. The series $\sum a_n$ is said to be *absolutely summable* (A), or *summable* $| A |$, if

$$F(x) = \sum a_n x^n$$

is convergent in $0 \leq x < 1$, and $F(x)$ is a function of bounded variation in $(0, 1)$ [14], [18].

Let $s_n^0 = s_n$ denote the n-th partial sum of the series $\sum a_n$, and let s_n^k and t_n^k denote the n-th Cesàro-means of order $k(k > -1)$ of the sequences $\{s_n\}$ and $\{na_n\}$ respectively. The series $\sum a_n$ is said to be absolutely summable (C, k), or summable $| C, k |$, if the sequence $\{s_n^k\}$ is of bounded variation, that is to say, the infinite series

$$\sum | s_n^k - s_{n-1}^k |$$

is convergent [5], [11].

The series $\sum a_n$ is said to be strongly summable (C, k), or summable $[C, k]$, $(k > 0)$, to sum s, if

$$\sum_{\nu=1}^{n} | s_\nu^{k-1} - s | = o(n),$$

as $n \to \infty$ [19].

In what follows we shall require the following well-known identities for $k > -1$.

$$t_n^k = n(s_n^k - s_{n-1}^k) \qquad [11], [12]; \tag{1.11}$$

$$t_n^{k+1} = (k + 1)(s_n^k - s_n^{k+1}) \qquad [7], [12]; \tag{1.12}$$

$$t_n^k = \frac{1}{A_n^k} \sum_{\nu=1}^{n} A_{n-\nu}^{k-1} \nu a_\nu , \tag{1.13}$$

where

$$A_n^k = \Gamma(n + k + 1)/\{\Gamma(n + 1)\Gamma(k + 1)\} \sim n^k/\Gamma(k + 1). \tag{1.14}$$

1.2. Let $f(t)$ be a periodic function with period 2π and integrable (L) over $(-\pi, \pi)$. Without loss of generality we may assume that the constant term in the Fourier series of $f(t)$ is zero and

$$f(t) \sim \sum (a_n \cos nt + b_n \sin nt) = \sum c_n(t), \tag{1.21}$$

$$\int_{-\pi}^{\pi} f(t)\, dt = 0. \tag{1.22}$$

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