GENERALIZED LOCAL CLASS FIELD THEORY. II. EXISTENCE THEOREM

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This is a continuation of [31], preserves its terminology, and carries on its numbering of everything. An adequate existence theorem—characterization of norm groups by conditions involving only the ground field k—hasn't been given before in the case of higher ramification which is the deep and interesting part of the theory. We prove such a theorem here by showing that a subgroup of $k^{\cdot}$ is norm group if and only if it has a conductor, has finite index, and contains a subgroup definable by polynomials in a certain way. (For exact definition see §5. Groups with these properties shall be called *analytic.*) Cardinal number and other considerations ([15], and work of ours to be published later) show that a simpler characterization could not be expected.

Our method is to prove by direct construction (§6) that there is a cyclic extension of degree p with any possible image groups (§4), to use this to reduce the problem of constructing an extension with given analytic group a as its norm group to the same problem for an a' of lower conductor, and to extend the theorem for groups with $k^{\cdot}/a$ cyclic of order p to that for $k^{\cdot}/a$ cyclic of degree p^m by the well known method of Chevalley [26]. Crucial lemmas are: *the intersection of two analytic groups is analytic*, and *the subgroup of an extension field whose norms fall into an analytic group is analytic.* We prove them both at once by a brief argument depending entirely on the theory of additive polynomials recently given by T. Crampton and ourselves [27], [32]. This is the application to class field theory promised in those papers.

Our proof (§6) of existence of fields whose norm group has prescribed images involves the problem of explicitly determining the conductor of a field from a defining equation. Much work has been done on this and problems related to it. Our approach (though independently found) closely resembles that of 0. Ore [29].

Changes in [30]: Denote algebraic closure now by k^{clos}, reserving "c" for something else. In Proposition 7 delete the term "$o(\pi^{m(i)})$". In Propositions 4, 5, 6, 7 delete assumption that $\bar{k}$ algebraically closed. Original proofs hold. Let $[k]$ denote $\bar{k}$ and $[\alpha]$ residue class of α mod π (ε $[k]$).

4. **Image groups.** Let k be a regular local field, $[k]$ its residue class field, (of characteristic $p \neq 0$) and π a fixed prime element. Our image groups will depend on choice of π. Define $k^{\cdot}_{(0)}$ to be the group of elements of value 1 and,

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