REFLEXITIVITY IN THE L^λ FUNCTION SPACES

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1. Let S denote an arbitrary space of points P with a countably additive, non-negative measure $\gamma(E)$ defined for a complemented, countably additive family of sets. As is well known, for $1 < p < \infty$ the Banach space L^p is reflexive; more generally, as shown by R. S. Phillips [5], for $1 < p < \infty$ and B an arbitrary reflexive Banach space, the space $L^p(B)$ is reflexive.

The L^p are included among the L^λ spaces introduced and studied in [2], although the L^p are not typical of the more general L^λ (see [3]). Nevertheless an extension of Phillips' result does hold; in the present paper, using the results of [2], we shall prove:

Theorem 1.1. *The space $L^\lambda(B)$ is reflexive if and only if both L^λ and B are reflexive.*

Theorem 1.2. *The space L^λ is reflexive if and only if the following conditions hold*:

(1.1) *Either S is coarse or the relations $\lambda(E) < \infty$, $e \subset E$, $\gamma(e) \to 0$ together imply* $\lambda(e) \to 0$.

(1.1)* *Either S is coarse or the relations $\lambda^*(E) < \infty$, $e \subset E$, $\gamma(e) \to 0$ together imply* $\lambda^*(e) \to 0$.

(1.2) *The finitely valued functions $f = \sum_i k_{e,i}$ with $\lambda(f)$ finite are dense in L^λ.*

(1.2)* *The finitely valued functions $f = \sum_i k_{e,i}$ with $\lambda^*(f)$ finite are dense in* $L^\mu(\mu = \lambda^*)$.

(1.3) $\lambda^*(G) < \infty$ *implies $G = G_E$ for some $E = e_1 + e_2 + \cdots$ with all $\lambda(e_i)$ finite.*

(1.3)* $\lambda^{**}(G) < \infty$ *implies that $G = G_E$ for some $E = e_1 + e_2 + \cdots$ with all $\lambda^*(e_i)$ finite.*

(1.4) *Every measurable function $u(P)$ can be expressed as $u = u_1 + u_2$ with* $\lambda^{**}(u) = \lambda^{**}(u_1) = \lambda(u_1)$ *and* $\lambda^{**}(u_2) = 0$.

Here we assume that the Banach space B contains at least one non-zero element and we use the following conventions: E denotes an arbitrary measurable set; the scalars may be the real or the complex numbers; for k an arbitrary scalar or element in B, the function on S with value k for all P is denoted as $k = k(P)$; k_E denotes the function with value k for $P \in E$ and value 0 for all other P; $k_{E,i}$ is an abbreviation for k_E when $k = k_i$ and $E = E_i$; $\lambda(E)$ is an

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