# IRREDUCIBILITY OF THE SUM OF THE ELEMENTS OF A CONTINUOUS COLLECTION OF CONTINUA

By Eldon Dyer

In 1935, B. Knaster showed (2) that there exists a *continuous* collection $G$ of mutually exclusive non-degenerate continua such that $G$ is an arc with respect to its elements and $G^*$ (the sum of all of the elements of $G$) is a compact *irreducible* continuum. He raised the question whether there exists a collection fulfilling these conditions together with the additional condition that all of its elements are topologically equivalent to each other.

In 1949, E. E. Moise showed (3) that if $G$ is a continuous collection of mutually exclusive arcs such that $G$ is an arc with respect to its elements, then $G^*$ is not a compact irreducible continuum. In 1951, M. E. Hamstrom showed (1) that if $G$ is a continuous collection of mutually exclusive, non-degenerate, continuous curves such that $G$ is an arc with respect to its elements, then $G^*$ is not a compact irreducible continuum.

In the present paper it will be shown that there does not exist a continuous collection of mutually exclusive, *decomposable* continua such that the sum of all of its elements is a compact irreducible continuum.

It is understood throughout this paper that space is metric and compact.

Definitions. If $H$ and $K$ are point sets, the notation $d(H, K)$ will be used to denote the greatest number such that no point of $H$ is at a distance less than that number from any point of $K$. For two such sets, the *separation number*, $S(H, K)$, is the greatest lower bound of the set of all numbers $e$ such that if $P$ is a point of one of the sets, then there is a point $Q$ of the other such that $d(P, Q)$ is less than $e$. If $H$, $K$ and $L$ are point sets, then $S(H, L) \leq S(H, K) + S(K, L)$. If $M_1, M_2, \cdots$ is a sequence of point sets and $M$ is a closed point set, then $M_1, M_2, \cdots$ converges to $M$ if and only if $S(M, M_1), S(M, M_2), \cdots$ converges to the number zero.

Lemma 1. *If $K$ is a closed point set, $M$ is a continuum, $A$ and $B$ are two points of $M$ and $e$ is a positive number such that each subcontinuum of $M$ containing $A$ and $B$ contains a point at a distance from $K$ of more than $e$, then there is a positive number $r$ such that if $a$ and $b$ are points of a continuum $m$ and $d(A, a)$, $d(B, b)$ and $S(M, m)$ are less than $r$, then each subcontinuum of $m$ containing $a$ and $b$ contains a point at a distance from $K$ of more than $e$.*

Definitions. Let $G$ denote a continuous collection of mutually exclusive continua.

If $k$ is a subcontinuum of an element $g$ of $G$ and $k$ is not a subset of the closure

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