ORBIT SPACES OF FINITE TRANSFORMATION GROUPS. I.

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1. **Introduction.** Let X be a compact finite-dimensional metric space, and let G be a finite transformation group on X. Let Y denote the space of orbits of X, where the topology is the usual decomposition topology. We shall be concerned with the problem of finding topological properties which are transmitted from X to Y. Although the problem for general G does not appear to have been studied previously, results have been obtained from various restricted classes of groups G. In the course of his work on fixed points of periodic maps, P. A. Smith [3] proved, for the case in which G is a cyclic group of prime order, that certain homological properties with the integers mod p as coefficients are transmitted from X to Y. Later, the author [1] proved that a group of properties are transmitted from X to Y in case G is solvable. Among these were the property of being homologically locally connected in all dimensions over the integers, and the property of being an absolute retract. Recently, Liao [2], in his work on fixed points of periodic maps, has discovered other such properties. One such property is that of possessing finitely generated integral Čech cohomology groups.

The purpose of the present paper is to extend some of the results for G solvable of our earlier paper [1] to the case of a finite group G operating simplicially on a finite complex X. We show that if X is (1) homologically trivial over the integers, or (2) an absolute retract; then so also is Y. In a later paper we shall use the expected limiting technique to obtain analagous results for finite transformation groups on compact finite-dimensional metric spaces. The method used is one of reduction of the general case to the solvable case. In turn, the solvable case was handled by reduction to the case in which G is cyclic of prime order.

2. **Finite groups of simplicial maps.** Let X be a compact Hausdorff space and suppose that G is a finite group of homeomorphisms on X. Consider a Hausdorff space Y and a continuous map f of X onto Y such that $f(x) = f(y)$ for $x, y \in X$ if and only if there exists $g \in G$ with $gx = y$. Then Y is an *orbit space* and f is an *orbit map* for the pair $\{X, G\}$.

2.1. *If G is a finite transformation group on a compact Hausdorff space X, then there exists an orbit space and an orbit map for $\{X, G\}$. If Y, f and $\bar{Y}$, $\bar{f}$ are two such sets, then there exists a homeomorphism h of Y onto $\bar{Y}$ with $hf = \bar{f}$. Moreover, an orbit map f is open.*

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