AN EXAMPLE OF A CONNECTED IRRESOLVABLE HAUSDORFF SPACE

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1. **Introduction.** The purpose of this note is to answer in the affirmative a question of E. Hewitt [1; 327] regarding the existence of connected irresolvable Hausdorff spaces. We begin by stating some useful definitions.

A space R has been defined by Hewitt to be *resolvable* if R is dense-in-itself and is the union of two disjoint dense subsets; *irresolvable* if R is dense-in-itself and not resolvable; an *SI-space* if R is dense-in-itself and no subset of R is resolvable; and an *MI-space* if R is dense-in-itself and every dense subset of R is open.

As Hewitt has remarked, an MI-space is also an SI-space, and an SI-space is irresolvable. Thus these properties are in increasing order of strength.

A set A contained in R is defined to be a *boundary set* if it has void interior; a *nondense* set if the closure of A, $\mathrm{Cl}(A)$, has void interior—this is the same as "nowhere dense" in the terminology of some writers; and a *totally isolated set* if A has no limit points in the space R.

In §2, a general method of forming an MI-expansion of an SI-space is pointed out. In §3, a connected MI-space is constructed which is also a Urysohn space (*i.e.* a space in which any two distinct points can be separated by closed disjoint neighborhoods). Since an MI-space is irresolvable and a Urysohn space is a Hausdorff space, this result implies more than the existence of connected irresolvable Hausdorff spaces—the question raised by Hewitt. In §4, the question of local connectedness of irresolvable Hausdorff spaces is considered. It is proved that a Hausdorff MI-space is not locally connected at any point.

All spaces considered in this are T_1-spaces.

2. **The MI-expansion of a given SI-space.** MI- and SI-spaces can be characterized by means of the families of their boundary sets, nondense sets, and totally isolated sets. These families, which are functions of the given topology t on R, are denoted by B_t, N_t and T_t respectively in the following. Hewitt has proved that in an SI-space, every boundary set is nondense, *i.e.*, $B_t \subset N_t$, and that in an MI-space, every nondense set is totally isolated, *i.e.*, $N_t \subset T_t$. These properties of a space or of a subspace under its relative topology, namely, that every boundary set is nondense, and that every nondense set is totally isolated, are denoted by P and Q respectively in the following. It is clear from the above that for a dense-in-itself space R, P implies that R is irresolvable and is implied by the statement that R is an SI-space.

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