ABSOLUTELY CONTINUOUS PRODUCT TRANSFORMATIONS OF THE PLANE

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1. **Introduction.** This paper is concerned with the product of plane transformations, that is, one plane transformation followed by another plane transformation. The transformations considered will always be continuous and bounded on bounded domains (see 2.1). More specifically, we shall be interested in such transformations which are also either absolutely continuous in the sense of Banach (see 2.5) or essentially absolutely continuous, that is, absolutely continuous in the sense of Radó (see 2.11).

The purpose of the paper is to find conditions under which the product transformation of two absolutely continuous plane transformations is again absolutely continuous. By means of an example we show that in general, both in the Banach sense and the essential sense, the product transformation is not absolutely continuous (see § 5).

Analogous to the derivative of a function of a single real variable, a derivative function is associated with every absolutely continuous plane transformation. We shall be concerned with the relation of the derivative of the product transformation to the derivatives of the factor transformations.

When each of the factor transformations is absolutely continuous in the sense of Banach, we have established the following characterization for absolutely continuous product transformations.

THEOREM. *The product of two plane transformations, each absolutely continuous in the sense of Banach, is absolutely continuous in the sense of Banach if and only if the product of the derivatives of the factor transformations is summable over the domain of the first transformation. Moreover, if the product transformation is absolutely continuous then its derivative is equal to the product of the derivatives of the factor transformations a.e. (almost everywhere) on the domain of the first transformation (see 4.2).*

A Lipschitzian transformation (see 4.3) is easily shown to be absolutely continuous in the sense of Banach. As a corollary to the above theorem we show that an absolutely continuous transformation in the sense of Banach followed by a Lipschitzian transformation yields a transformation which is absolutely continuous in the sense of Banach (*see* 4.4).

Concerning essentially absolutely continuous transformations we have the result: