NORMAL EQUATIONS AND RESOLVENTS IN FIELDS OF CHARACTERISTIC p

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1. **Introduction.** The problem considered in this paper is that of reducing equations of degree 5, with coefficients in a field F of characteristic p, where $p = 2, 3, 5$, to so-called *principal* and *normal* forms by means of Tschirnhaus transformations. In addition, a sextic resolvent is obtained in each case. For corresponding results in the case of F of characteristic zero see [2; Chapter XII]. It is remarked that the reduction of the quintic in [2; Chapter XII] applies when the characteristic of F is zero or $p \geq 7$.

2. **Preliminary.** Let $x_1, x_2, \cdots, x_n$ be the roots of the equation

$$f(x) = x^n + a_1x^{n-1} + \cdots + a_n = 0 \qquad (a_i \in F), \tag{2.1}$$

where the characteristic of the field F is a prime p. A Tschirnhaus transformation (see [3; Chapter XII], [4; vol. 2, Chapter V], [5], [7; Chapter XVIII], [8; §II, Chapter I]) of (2.1) into

$$f(y) = y^n + b_1y^{n-1} + \cdots + b_n = 0 \tag{2.2}$$

is taken in the form

$$y = cx^{n-1} + dx^{n-2} + \cdots + f. \tag{2.3}$$

The coefficients of the new equation (2.2) are determined, by use of symmetric functions, in terms of the coefficients of (2.3) which may be regarded as parameters.

For later reference, Newton's identities [1; Chapter VIII] for the sums of powers of the roots of (2.1) are given generally by

$$s_k = -a_1s_{k-1} - a_2s_{k-2} - \cdots - a_{k-1}s_1 - ka_k, \tag{2.4}$$

where the s_k denote the sums of the k-th power of the roots $x_1, x_2, \cdots, x_n$ of (2.1) in terms of the coefficients a_i. In particular (2.4) implies

$$\begin{gathered} s_1 = -a_1, \qquad s_2 = a_1^2 - 2a_2, \qquad s_3 = -a_1^3 + 3a_1a_2 - 3a_3, \\ s_4 = a_1^4 - 4a_1^2a_2 + 2a_2^2 + 4a_1a_3 - 4a_4. \end{gathered} \tag{2.5}$$

The notation $\sum y$ will denote $\sum_{i=1}^n y(x_i)$, the x_i being the n roots of (2.1). $\sum y$ is obtained directly by summing (2.3) for the roots of (2.1) by use of (2.5).

The term *principal* equation [6] will be applied to (2.1) with $a_1 = a_2 = 0$. Similarly, the term *normal* form will generally apply to the equation (2.1) in

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