GENERALIZED INVERSION FORMULAS FOR CONVOLUTION TRANSFORMS, II

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1. Introduction. The authors have discussed in [5] and elsewhere the inversion of the convolution transform

$$f(x) = \int_{-\infty}^{\infty} G(x - t)\phi(t)\,dt \tag{1}$$

for a large class of kernels $G(t)$. In order to obtain a new type of inversion, which is itself a convolution,

$$\phi(t) = (1/2\pi i)\int_C f(t + z)K(z)\,dz, \tag{2}$$

we propose to limit somewhat the class of kernels $G(t)$. The kernels to be considered are

$$G(t) = (1/2\pi i)\int_{-i\infty}^{i\infty} e^{st}/E(s)\,ds \tag{3}$$

where

$$E(s) = \prod_{k=1}^{\infty} (1 - s^2 a_k^{-2}), \tag{4}$$

the a_k being real and such that

$$\lim_{k\to\infty} a_k/k = 1. \tag{5}$$

Thus if $a_k = (2k - 1)/2$, $E(s) = \cos \pi s$ and $G(t) = (2\pi)^{-1} \operatorname{sech} \frac{1}{2}t$. Setting $f(x) = F(e^x)e^{x/2}$ and $\phi(t) = \pi\Phi(e^t)e^{t/2}$, equation (1) becomes the Stieltjes transform

$$F(x) = \int_0^{\infty} \Phi(t)/(x + t)\,dt. \tag{6}$$

The classical complex inversion of [6] due to Stieltjes is

$$\Phi(t) = \lim_{\epsilon\to 0+} (1/2\pi i)[F(-t - i\epsilon) - F(-t + i\epsilon)] \qquad (0 < t < \infty). \tag{7}$$

Our inversion (2) will include (7) as a special case.

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