THE VIRTUAL MASS OF NEARLY SPHERICAL SOLIDS

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1. **Introduction.** 1.1. We consider a solid moving in an infinite fluid which is assumed to be incompressible, frictionless and irrotational. The uniform density of the fluid should be denoted by ρ. We assume that it is at rest at an infinite distance.

As to the solid we assume that its motion is a pure translation. Its velocity is characterized by a vector $\mathbf{h}U$ where $\mathbf{h}$ is a unit vector in a coordinate system x_1, x_2, x_3 rigidly connected with the solid, and $U > 0$. Denoting the kinetic energy of the fluid by T, the equation

$$T = \tfrac{1}{2}WU^2 \tag{1}$$

defines a quantity W which is called the *virtual mass* (added mass). The kinetic energy can be computed by using the velocity potential $\phi = \phi(x_1, x_2, x_3)$. This is a function harmonic outside of the solid, satisfying the boundary condition

$$-\partial\phi/\partial n = h_n, \tag{2}$$

where n is the exterior normal and h_n the projection of the unit vector $\mathbf{h}$ onto this normal. The function ϕ has the same behavior at infinity as the potential of a dipole. We have then

$$T = \tfrac{1}{2}\rho U^2 \iiint |\operatorname{grad} \phi|^2 \, dx_1 \, dx_2 \, dx_3, \tag{3}$$

the integration being extended over the exterior of the solid. Consequently

$$W/\rho = \iiint |\operatorname{grad} \phi|^2 \, dx_1 \, dx_2 \, dx_3 = -\iint \phi(\partial\phi/\partial n) \, d\sigma = \iint \phi h_n \, d\sigma; \tag{4}$$

the two last integrals are extended over the surface of the solid.

The ratio W/ρ is an important quantity depending on the shape and size of the given solid, of the choice of the coordinate system x_1, x_2, x_3 therein and of the velocity vector $\mathbf{h}$. More precisely, it is a *quadratic form* of the direction cosines h_1, h_2, h_3 of $\mathbf{h}$.

1.2. Assuming $\rho = 1$ we can write for the virtual mass

$$W = \sum_{i,k=1,2,3} W_{ik} h_i h_k \qquad (W_{ik} = W_{ki}). \tag{5}$$

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