THE KERNEL FUNCTION AND CANONICAL CONFORMAL MAPS

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1. One of the fundamental problems in the theory of conformal mapping is the construction of analytic functions effecting the conformal mapping of given multiply-connected domains D onto certain types of canonical domains. The classical solutions of this problem (see [6] and papers quoted therein) are either pure existence proofs, or else construct the mappings in question with the help of certain domain functions—such as the Green's function, the harmonic measures, *etc.*—and their periods, which are supposed to be known.

While satisfactory from a theoretical point of view, these constructions are hardly of practical value, since the computation of the required domain functions presents, in the general case, very considerable difficulties. There exists, on the other hand, a domain function which can be computed by means of a simple algorithm, namely the kernel function $K(z, \zeta^*)$, introduced by Bergman [1]. (Complex conjugates are denoted by asterisks.) For the purpose of effectively constructing the canonical mapping functions it is therefore desirable to obtain representations of these functions in terms of the Bergman kernel function.

Before going into details, we mention here those properties of the kernel function $K(z, \zeta^*)$ of a finite domain D which will be needed in the sequel [2], [3].

(1) $K(z, \zeta^*)$ $(z \varepsilon D, \zeta \varepsilon D)$ is regular in D and possesses there a single-valued integral $T(z, \zeta^*) = \int_\zeta^z K(z, \zeta^*)\, dz$;

(2) $K(z, \zeta^*)$ is Hermitian, *i.e.*, $K(\zeta, z^*) = (K(z, \zeta^*))^*$;

(3) $K(z, \zeta^*)$ has the reproducing property

$$\iint_D (K(z, \zeta^*))^* f(z)\, d\sigma = f(\zeta) \qquad (d\sigma = dx\, dy;\, z = x + iy), \tag{1}$$

with regard to any function $f(z)$ which is regular and of class L^2 in D and possesses there a single-valued integral. Conversely, properties (1)–(3) determine the Bergman kernel function in a unique manner.

If the boundary Γ of D consists of analytic curves and $f(z)$ is regular on Γ, (1) may be written in a different form. Using Green's identity

$$\iint_D pq^{*\prime}\, d\sigma = \frac{1}{2i} \int_\Gamma pq^*\, dz, \tag{2}$$

Received May 27, 1948. Paper done under contract with the Office of Naval Research (N5ori 76–16–Nr 043–046).