THE LOWER BOUND OF THE ORDER OF A PRODUCT SET OF POLYNOMIALS

By M. T. Eweida

1. **Introduction.** A set of polynomials $p_0(z), p_1(z), \cdots, p_n(z), \cdots$ is said to be basic, if any arbitrary polynomial can be expressed in one and only one way as a finite linear combination of them.

Let

$$z^n = \pi_{n0}p_0(z) + \pi_{n1}p_1(z) + \cdots, \tag{1.1}$$

$$\omega_n(R) = |\pi_{n0}| M_0(R) + |\pi_{n1}| M_1(R) + \cdots, \tag{1.2}$$

where

$$M_n(R) = \max_{|z|=R} |p_n(z)|, \tag{1.3}$$

$$\omega = \overline{\lim_{n\to\infty}} (\log \omega_n(R))/n \log n; \tag{1.4}$$

ω is said to be the order of the basic set of polynomials $\{p_n(z)\}$.

If $f(z)$ is a function regular at $z = 0$, then the series

$$p_0(z)\Pi_0 f(0) + p_1(z)\Pi_1 f(0) + \cdots, \tag{1.5}$$

where

$$\Pi_n f(0) = \pi_{0n}f(0) + \pi_{1n}\frac{f'(0)}{1!} + \pi_{2n}\frac{f''(0)}{2!} + \cdots$$

and $p_n(z) = p_{n0} + p_{n1}z + p_{n2}z^2 + \cdots$, is called the basic series associated with the given basic set of polynomials.

Also, the matrix

$$P \equiv (p_{ij}) \qquad (i, j = 0, 1, \cdots)$$

is said to be the matrix of coefficients, and the matrix

$$\Pi \equiv (\pi_{ij}) \qquad (i, j = 0, 1, \cdots)$$

is the matrix of operators.

Whittaker [3] proved the following results:

(a) The necessary and sufficient condition for a set of polynomials to be basic is $(P\Pi) = 1$. ($(P\Pi)$ is the product of the two matrices (P) and (Π) in the usual way.)

Received April 14, 1948. The author wishes to express his thanks to Professor J. M. Whittaker, the supervisor of this work, for his valuable help and guidance during the past two years in Liverpool University.