EXPRESSING A FUNCTION OF THREE VARIABLES IN NOMOGRAPH FORM

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1. The first step in constructing a nomograph is to express a given function of x, y and z in the form of a three-rowed determinant whose first, second and third rows are functions of x alone, y alone and z alone, respectively:

$$f(x, y, z) \equiv \begin{vmatrix} X_1(x) & X_2(x) & X_3(x) \\ Y_1(y) & Y_2(y) & Y_3(y) \\ Z_1(z) & Z_2(z) & Z_3(z) \end{vmatrix}. \tag{1}$$

It is the purpose of this article to give necessary and sufficient conditions that a function can be expressed in this *nomograph form,* and formulas for so expressing it. We shall state the results first, and then outline the proofs.

Duporcq [1] gives necessary and sufficient conditions that a function may be expressed in nomograph form, and suggests a formula for so expressing it. No derivatives are used, but instead three sets of constant values of the variables are used in various combinations with the variables themselves. Kellogg [3] also gives necessary and sufficient conditions but no formula for expressing a function in nomograph form. He employs derivatives, and in particular his conditions (7) with the modifying statement in the following paragraph, and (26) and (32) are very similar indeed to the conditions given in the present paper. He loses somewhat in elegance, however, by giving a special role to one of the variables. Moreover his (32) is unnecessarily long. It may be shown that when his (26) is satisfied, the second and fourth terms of his (32) cancel and so might be left out.

2. Theorem 1. (*The function f and all its partial derivatives are supposed to have the arguments x, y, z which are omitted for brevity.*) *In the general case where*

$$f(ff_{xy} - f_xf_y)(ff_{xz} - f_xf_z)(ff_{yz} - f_yf_z) \not\equiv 0, \tag{2}$$

necessary and sufficient conditions that the function $f(x, y, z)$ be expressible in nomograph form are

$$\begin{vmatrix} f & f_y & f_{yy} \\ f_x & f_{xy} & f_{xyy} \\ f_{xx} & f_{xxy} & f_{xxyy} \end{vmatrix} \equiv 0, \tag{3}$$

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