COUNTABLE ABELIAN GROUPS WITHOUT TORSION

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The torsion $\mathfrak{T}$ of an infinite Abelian group $\mathfrak{G}$ is defined as the subgroup formed by all the elements of finite order. The quotient group $\mathfrak{A} = \mathfrak{G}/\mathfrak{T}$ is torsion-free, *i.e.*, has no elements of finite order besides 0. Since the structure of Abelian torsion groups is settled by the Ulm theorem at least in the countable case [5], it is clear that the study of torsion-free groups is of primary importance for the general theory of infinite Abelian groups.

This paper is devoted to the problem of countable torsion-free groups and their structure will be cleared up to a great extent. In generalizing ideas of A. Kurosch [2] who has used p-adic whole numbers and transformations to describe primitive torsion-free groups of finite rank (the idea seems to originate from F. Levi [3]), I will derive from the structure of the groups certain integral and p-adic invariants which will enable us to construct and characterize all the countable (primitive or imprimitive) torsion-free Abelian groups of finite or infinite rank.

The solution of the problem is still not complete insofar as no satisfactory criterion has been found to decide upon the equivalence of different systems of invariants, *i.e.*, under what conditions do they represent isomorphic groups?

Part of the results of A. Kurosch will appear here in a somewhat different form. For sake of completeness, I will prove my statements—with the exception of some elementary facts—even if they are essentially contained in Kurosch's work.

1. The Abelian group $\mathfrak{A}$ is called torsion-free if there are no elements of finite order in $\mathfrak{A}$ besides 0.

The elements $C_1, C_2, \cdots, C_k$ of $\mathfrak{A}$ are linearly dependent (or simply, dependent) if there is a relation

$$a_1C_1 + \cdots + a_kC_k = 0$$

with $a_i \neq 0$ for at least one of the coefficients. Otherwise, they are called independent.

We say, the element B is divisible by n, $B \equiv 0 \pmod{n}$, if $nX = B$ has a solution X in A. X is uniquely determined by this equation and can be written as $X = n^{-1}B$.

Let $C_1, C_2, \cdots, C_k$ be independent elements. They are called linearly dependent modulo p^n, if

$$(1) \qquad \alpha_1C_1 + \cdots + \alpha_kC_k \equiv 0 \pmod{p^n}$$

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