CERTAIN CHAINS IN A FINITE PROJECTIVE GEOMETRY

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1. **Introduction.** It is well known that a point in a finite projective plane $PG(2, p^n)$ may be denoted by the symbol (x_1, x_2, x_3), where the coordinates x_1, x_2, x_3 are elements of a Galois field of order p^n, $GF(p^n)$. The symbol $(0, 0, 0)$ is excluded, and the symbols (x_1, x_2, x_3) and (kx_1, kx_2, kx_3) are to be thought of as the same point, if k is a non-zero element of the $GF(p^n)$. The totality of points whose coordinates satisfy the equation $u_1x_1 + u_2x_2 + u_3x_3 = 0$, where u_1, u_2, u_3 are elements of the $GF(p^n)$, not all zero, is called a line. The plane then consists of $p^{2n} + p^n + 1$ points and $p^{2n} + p^n + 1$ lines; each line contains $p^n + 1$ points and through each point pass $p^n + 1$ lines. (See [3; 244].)

A collineation is defined as a one-to-one transformation carrying points into points and lines into lines. In the plane $PG(2, p^n)$ any transformation of the form

$$(1) \qquad x_i' = a_{i1}x_1^{p^r} + a_{i2}x_2^{p^r} + a_{i3}x_3^{p^r} \qquad (i = 1, 2, 3; r = 0, 1, \cdots, n-1),$$

where a_{ij}, $i, j = 1, 2, 3$, are elements of the $GF(p^n)$ and $| a_{ij} | \neq 0$, is a collineation (see [3; 252]), and conversely (see [2]).

If a set containing $p^{2r} + p^r + 1$, $0 < r \leq n$, points of the $PG(2, p^n)$ forms a finite projective plane $PG(2, p^r)$, it is called a subplane $PG(2, p^r)$ of the $PG(2, p^n)$.

Definition. A set of $p^r + 1$, $0 < r \leq n$, points on a line of the $PG(2, p^n)$ is called an r-ple chain, when the set is contained in a subplane $PG(2, p^r)$ of the $PG(2, p^n)$.

In particular, a simple chain corresponds to the chain defined by von Staudt ([1; 137]; see also [3; 249]).

The object of the present note is to discuss the chains of multiplicity $r = 2^m$ in a line. It will be noted that almost all the results for von Staudt's chains remain valid.

Before we proceed further, it is convenient to state here two useful lemmas.

Lemma 1. *Any three collinear points determine a unique simple chain.*

Lemma 2. *Every collineation carries an r-ple chain into an r-ple chain.*

2. **Chains whose multiplicities are powers of** 2. For simplicity, we choose $x_2 = 0$ as the equation of the line, and denote the abscissa $x_1 : x_3$ by x; $x = \infty$ if $x_3 = 0$.

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