SUMMABILITY FACTORS OF FOURIER SERIES

BY MIN-TEH CHENG

1. Let $f(x)$ be integrable in the sense of Lebesgue, periodic with period 2π, and let

$$f(x) \sim \tfrac{1}{2}a_0 + \sum_{n=1}^{\infty} (a_n \cos nx + b_n \sin nx) \equiv \sum_{n=0}^{\infty} A_n(x) \tag{1.1}$$

be its Fourier series. The conjugate series of (1.1) can be then written as

$$\sum_{n=1}^{\infty} (b_n \cos nx - a_n \sin nx). \tag{1.2}$$

A series $\sum_{n=0}^{\infty} a_n$ is said to be absolutely summable (A), or summable $| A |$, if

$$f(x) = \sum_{n=0}^{\infty} a_n x^n$$

is of bounded variation in the interval (0, 1); it is said to be absolutely summable (C, α), $\alpha > -1$, or summable $| C, \alpha |$, if the series

$$\sum | \sigma_n^\alpha - \sigma_{n-1}^\alpha |$$

converges, where σ_n^α denotes the n-th Cesàro mean of order α of the series

$$\sum_{n=0}^{\infty} a_n ,$$

that is,

$$\sigma_n^\alpha = \frac{1}{(\alpha)_n} \sum_{k=0}^{n} (\alpha)_k a_{n-k} \qquad ((\alpha)_k = \Gamma(k + \alpha + 1)/\Gamma(k + 1)\Gamma(\alpha + 1)).$$

If $\sum a_n$ is summable $| C, \alpha |$ for some α, we say that $\sum a_n$ is summable $| C |$.

It is known (see [2]) that a series is summable $| A |$, if it is summable $| C |$. J. M. Whittaker [7] proved that the series

$$\sum_{n=1}^{\infty} (a_n \cos nx + b_n \sin nx)/n^\alpha \qquad (\alpha > 0) \tag{1.3}$$

is summable $| A |$ almost everywhere. B. N. Prasad [7] improved this result by showing that the Fourier series (1.1) and its conjugate series (1.2) when multiplied by one of the following factors

$$\begin{gathered} 1/(\log n)^{1+\epsilon}, \qquad 1/(\log_1 n)(\log_2 n)^{1+\epsilon}, \qquad \cdots, \\ 1/(\log_1 n)(\log_2 n) \cdots (\log_{k-1} n)(\log_k n)^{1+\epsilon}, \end{gathered} \tag{1.4}$$

Received March 4, 1947.