# NOTE ON THE STRONG SUMMABILITY OF FOURIER SERIES

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1. Let $p$ be a number not less than 1. Let $f(u) \varepsilon L^p(-\pi, \pi)$ and be periodic with period $2\pi$. Let $S_n(u)$, $n = 0, 1, 2, \cdots$, denote the $n$-th partial sum of the Fourier series of $f$. Let $x$ be a fixed point, and let

$$S_n = S_n(x), \qquad \phi(u) = \tfrac{1}{2}\{f(x+u) + f(x-u) - 2S\}.$$

Then, if

$$(1.1) \qquad 1 < p, \qquad \int_0^t |\phi(u)|^p \, du = o(t) \qquad (t \to +0),$$

it follows that

$$(1.2) \qquad \sum_{m=0}^{n} | S_m - S |^r = o(n) \qquad (n \to \infty)$$

for every positive $r$ (see [2], [1], [6]). Further, Hardy and Littlewood [3] have shown that, if

$$(1.3) \qquad p = 1, \qquad \int_0^t |\phi(u)| \, du = o(t),$$

then

$$\sum_{m=0}^{n} | S_m - S |^r = \begin{cases} o[n(\log n)^{r/2}] & (0 < r \leq 2), \\ o[n(\log n)^{r-1}] & (2 < r). \end{cases}$$

The author [4] has proved that, if (1.3) holds, then

$$\sum_{m=0}^{n} | S_{m^k} - S |^2 = o(n \log n)$$

for every integer $k \geq 2$. The author [5] has also proved that, if (1.1) holds, then

$$(1.4) \qquad \sum_{m=0}^{n} | S_{m^k} - S |^2 = o(n)$$

for every integer $k$ such that $2 \leq k < p$. The object in this note is to extend this last result. We prove the following theorem.

Theorem. *Let the conditions* (1.1) *hold. Then* (1.4) *holds for every integer* $k \geq 2$.

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