THE CHORDAL HYPERSURFACES OF A RATIONAL CURVE

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1. The totality of k-spaces which are $(k + 1)$-secants of a rational curve in a p-dimensional space $(p > 2k + 1)$ form an algebraic variety V_{2k+1}^n of dimension $2k + 1$ and order n. The present paper is concerned with the study of some properties of this variety and with its representation upon a linear $(2k + 1)$-space.

The topic of chordal varieties bears a close relationship to several important geometrical subjects such as line complexes, Cremona n-ic transformations, and systems of quadrics in multi-dimensional spaces. There are also interesting connections with the theory of invariants, as pointed out by P. H. Schoute [5]. Some of the properties of V_{2k+1}^n and its relation to other geometrical topics have been investigated by B. Levi [2], A. Tanturri [7], H. Telling [8], and C. Segre [6]. More recently T. G. Room [4; §11.7] has discussed the subject briefly in connection with his work on determinantal loci.

2. A rational curve C^r of order r in a space S_p of dimension $p < r$ and its associated $(k + 1)$-secant k-spaces may be regarded as the projection of a rational, normal curve of order r in an r-space S_r together with its corresponding multisecant spaces. Without loss of generality we may then restrict our attention to the secant loci connected with a normal, rational curve C^r in S_r. The curve C^r may be represented parametrically in the form

$$x_0 : x_1 : x_2 : \cdots : x_r = 1 \quad t : t^2 : \cdots : t^r, \tag{2.1}$$

from which it is evident that C^r lies on the $\binom{r}{2}$ linearly independent quadric hypersurfaces obtained by equating to zero all the second order determinants of the matrix

$$\left\| \begin{matrix} x_0 & x_1 & x_2 & \cdots & x_{r-2} & x_{r-1} \\ x_1 & x_2 & x_3 & \cdots & x_{r-1} & x_r \end{matrix} \right\|. \tag{2.2}$$

This matrix puts in evidence two well-known projective generations of C^r: (see [4; 219]) as the intersection of the r projectively related pencils of hyperplanes given by

$$x_0 + tx_1 = 0, \qquad x_1 + tx_2 = 0, \quad \cdots, \quad x_{r-1} + tx_r = 0, \tag{2.3}$$

and as the locus of the points of intersection of corresponding lines determined by sets of corresponding hyperplanes of the two systems

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