SOME RESULTS ON DOUBLE FOURIER SERIES

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Introduction. This paper contains some of the results proved by us in the course of our attempts to develop fully the method, inaugurated by S. Bochner, of summing multiple Fourier series by spherical means [2]. After a few introductory remarks on notations and formulae in §1, we state some simple results on the Fourier coefficients in §2. In §3, we have collected some results on summability, which are direct consequences of Bochner's fundamental formula; and using these results, we study the problem of convergence in §4. The convergence tests are essentially Tauberian in nature, and include incidentally a test like Hardy-Littlewood's. §5 deals with the properties of circular averages of partial sums. We actually generalize Rogosinski's theorem [9; Theorem 87] and a result of Bernstein [1] on simple Fourier series of functions belonging to Lipschitz class. In §6, we give a new proof of a known theorem on absolute convergence (Theorem 6.1).

Most of these results can be extended to Fourier series in several dimensions (see Remarks in §7), but we have restricted ourselves to double Fourier series in order to keep the treatment relatively simple.

1. **Generalities.** Let $f(x, y)$ be a real-valued integrable function periodic with period 2π in each of the variables, and let us write, as usual,

$$f(x, y) \sim \sum_{p,q}^{-\infty \cdots +\infty} c_{pq} e^{i(px+qy)}, \tag{1.1}$$

$$c_{pq} = \frac{1}{4\pi^2} \int_{-\pi}^{\pi} \int_{-\pi}^{\pi} f(\xi, \eta) e^{-i(p\xi+q\eta)} \, d\xi \, d\eta, \tag{1.2}$$

the series on the right of (1.1) being called the double Fourier series of $f(x, y)$.

If we set

$$c_{00} = \tfrac{1}{2} a_0, \qquad 2c_{pq} = a_{pq} - i b_{pq}, \qquad 2c_{-p,-q} = a_{pq} + i b_{pq},$$

where $p > 0$ and q is arbitrary, then from (1.1) and (1.2) we have

$$f(x, y) = \tfrac{1}{2} a_{00} + \sum_{0 \le p < \infty}^{-\infty < q < \infty}{}' \{a_{pq} \cos(px + qy) + b_{pq} \sin(px + qy)\}, \tag{1.3}$$

Received February 13, 1947; presented to the American Mathematical Society, April, 26, 1947.