SUMS OF AN EVEN NUMBER OF SQUARES IN $GF[p^n, x]$, II

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1. Introduction. In [2] we considered the following problem: Let F be a polynomial in $GF[p^n, x]$, $p > 2$, of degree less than $2k$, where k is a fixed non-negative integer, and let $\alpha_1, \cdots, \alpha_{2s}$ be elements of $GF(p^n)$ such that

$$\alpha_1 + \cdots + \alpha_l = 0 \qquad (1 \leq l \leq 2s). \tag{1.1}$$

It was then required to find the number of solutions of

$$F = \alpha_1 X_1^2 + \cdots + \alpha_{2s} X_{2s}^2 \tag{1.2}$$

in polynomials $X_1, \cdots, X_{2s}$ where $X_1, \cdots, X_l$ are primary of degree k and $X_{l+1}, \cdots, X_{2s}$ are arbitrary of degree less than k. The solution of this problem was obtained for all values of l except the case excluded in (1.1), namely $l = 0$. It is the purpose of the present paper to treat this remaining case. In other words, we want the number of solutions of (1.2) in polynomials of degree less than k under the single restriction that F is of degree less than $2k$.

We now define certain divisor functions used in this paper. Let M represent a polynomial of $GF[p^n, x]$. Then we define

$$\delta_z(M) = \begin{cases} \displaystyle\sum_{Z \mid M}^{\deg Z = z} 1 & (z \geq 0), \\ 0 & (z < 0), \end{cases} \tag{1.3}$$

where the summation is over primary Z only; in particular

$$\delta_z(0) = \begin{cases} p^{nz} & (z \geq 0), \\ 0 & (z < 0). \end{cases}$$

Further, we place

$$g_z(M) = \delta_z(M) - \delta_{2k-z-1}(M), \tag{1.4}$$

and finally we define $R_s(M, \mu)$ by

$$R_s(M, \mu) = \{p^{n(s+1)} - \mu\} p^{ns(2k-1)} \sum_{z=0}^{k-1} \mu^z p^{-nzs} g_z(M) \tag{1.5}$$

when $M \neq 0$, and

$$\begin{aligned} R_s(0, \mu) &= p^{2kn(s+1)} \\ &\quad + \{p^{n(s+1)} - \mu\} p^{ns(2k-1)} \sum_{z=0}^{k-1} \mu^z p^{-nzs} g_z(0). \end{aligned} \tag{1.6}$$

Received April 29, 1947.