THE PROJECTIVE DEFORMATION OF NON-HOLONOMIC SURFACES

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1. **Introduction.** The deformation of ordinary surfaces has been one of the important problems in metric differential geometry. It can be regarded as a point correspondence between two surfaces S and $\bar{S}$ such that to each pair of corresponding points A and $\bar{A}$, of S and $\bar{S}$ respectively, there exists a rigid motion carrying A to $\bar{A}$, and at the same time carrying the neighborhood of the first order of A to that of $\bar{A}$. (See [2].) It is the aim of the present paper to generalize this notion to the non-holonomic surfaces in projective space and study some of the properties of the generalization.

Let us consider a non-holonomic surface S in the ordinary projective space. A point together with the tangent plane at the point (see [1]) form an element of contact so that S defines a three-parameter family F of elements of contact. Since the surface is non-holonomic there exists no two-parameter sub-family of F having the property that the planes are tangent to the surface described by the corresponding origins of the elements of contact. Conversely, given any such family F^* of elements we can construct a non-holonomic surface S^* related to F^* as S to F. Hence we may regard a non-holonomic surface as a three-parameter family of elements of contact. From this point of view it is natural to lay down the following definition (see [3]):

A correspondence of the elements of two non-holonomic surfaces S and $\bar{S}$ is called a projective deformation if to each pair of corresponding elements E and $\bar{E}$, of S and $\bar{S}$ respectively, there exists a projective transformation carrying E to $\bar{E}$ and carrying the neighborhood of the first order of E to that of $\bar{E}$.

We can also state it as follows:

An automorphism $P \leftrightarrow \bar{P}$ of the space is called a projective deformation between two non-holonomic surfaces S and $\bar{S}$ if to each pair of corresponding points A and $\bar{A}$ there exists a projective transformation $T(A)$ such that (a) it carries A to $\bar{A}$ and the tangent plane of S at A to that of $\bar{S}$ at $\bar{A}$; and (b) on neglecting infinitesimals of orders higher than the first, it also carries any neighboring point A' of A to the corresponding point $\bar{A}'$, and the tangent plane of S at A' to that of $\bar{S}$ at $\bar{A}'$.

2. **Ordinary frames; associated projectivity.** A set of four analytic points $AA_1A_2A_3$ satisfying the condition

$$(1) \qquad | AA_1A_2A_3 | = 1$$

is called a *projective frame* [2]. To study the intrinsic property of a non-holonomic surface S we attach to each point of the space the family of all the pro-

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