PLANE SECTIONS OF THE TANGENT SURFACES OF TWO SPACE CURVES

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1. Introduction. Let C, $\bar{C}$ be two curves in ordinary space having P for their common point with distinct tangents t, $\bar{t}$ but the same osculating plane. If a general plane π intersects the tangents t, $\bar{t}$ respectively at points Q, $\bar{Q}$, then the plane sections Γ, $\bar{\Gamma}$ of the tangent surfaces of the curves C, $\bar{C}$ made by π have a common tangent at Q, $\bar{Q}$. In a recent paper the author [2] called such points Q, $\bar{Q}$ *ordinary points of the second kind* of the plane sections Γ, $\bar{\Gamma}$, for which several osculants have been introduced. The purpose of the present paper is to study these osculants for the plane sections Γ, $\bar{\Gamma}$ and also to give an application to the asymptotic curves through an ordinary point of a surface.

§2 contains power series expansions of the plane sections Γ, $\bar{\Gamma}$ which are used in later developments. In §3, we find the loci of two particular osculants of the plane sections Γ, $\bar{\Gamma}$ as the plane π revolves about the line $Q\bar{Q}$, and especially arrive at a certain correspondence. In the last section we apply the results obtained in §3 to the interesting case where C, $\bar{C}$ are asymptotic curves of a surface, and then derive a new geometrical characterization of the second projective normal of the surface at the point P.

2. Power series expansions. Let C, $\bar{C}$ be two curves in ordinary space having P for their common point with distinct tangents t, $\bar{t}$ but the same osculating plane $(t, \bar{t})$. If t, $\bar{t}$ be taken as the axes x, y, then the power series expansions of the two curves C, $\bar{C}$ in the neighborhood of the point P may be respectively written in the form

$$y = ax^2 + bx^3 + \cdots, \qquad z = rx^3 + sx^4 + \cdots; \tag{1}$$

$$x = \alpha y^2 + \beta y^3 + \cdots, \qquad z = \rho y^3 + \sigma y^4 + \cdots. \tag{2}$$

The tangents of the curve C describe a developable surface T, namely, the tangent surface. The equations of the tangent surface T are evidently of the form

$$\xi = x + \mu, \qquad \eta = y + \mu y', \qquad \zeta = z + \mu z', \tag{3}$$

where $y' = dy/dx$, $z' = dz/dx$; μ denotes another parameter and ξ, η, ζ the current coordinates of a point.

Let us consider a general plane π which does not pass through the point P:

$$\zeta = \lambda(C + B\xi + A\eta) \qquad (\lambda C \neq 0). \tag{4}$$

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