THE COMPOSITION OF CUBIC FORMS

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The recent work by L. E. Dickson [1; 268] and C. C. MacDuffee [2] may be extended by introducing a specified form f.

Notation. Let R be a commutative and associative ring with a modulus 1, and let $f(x) = \sum_i X_i Y_i Z_i$ be a cubic form, X_i, Y_i, Z_i being linear in the indeterminates $x_1, x_2, \cdots, x_n$ over R. Let

$$P_1 = \begin{vmatrix} X_1 & X_2 & 0 \\ 0 & Y_1 & Y_2 \\ Z_2 & 0 & Z_1 \end{vmatrix}, \quad U_2 = \begin{vmatrix} P_1 & 0 \\ 0 & P_1 \end{vmatrix}, \quad V_2 = \begin{vmatrix} P_1 - X_1 & P_1 - X_1 - Y_1 \\ Z_1 & P_1 - Y_1 \end{vmatrix},$$

$$W_2 = \begin{vmatrix} P_1 - Y_1 & -P_1 + X_1 + Y_1 \\ -Z_1 & P_1 - X_1 \end{vmatrix}, \qquad P_i = \begin{vmatrix} U_i & X_{i+1} & 0 \\ 0 & V_i & Y_{i+1} \\ Z_{i+1} & 0 & W_i \end{vmatrix},$$

$$U_{i+1} = \begin{vmatrix} P_i & 0 \\ 0 & P_i \end{vmatrix}, \quad V_{i+1} = \begin{vmatrix} P_i - U_i & P_i - U_i - V_i \\ W_i & P_i - V_i \end{vmatrix},$$

$$W_{i+1} = \begin{vmatrix} P_i - V_i & -P_i + U_i + V_i \\ -W_i & P_i - U_i \end{vmatrix};$$

where $P_i - U_i$ is interpreted as

$$\begin{vmatrix} 0 & X_{i+1} & 0 \\ 0 & V_i - U_i & Y_{i+1} \\ Z_{i+1} & 0 & W_i - U_i \end{vmatrix},$$

each of the constituent matrices being of the same order. Hence U_i, V_i, W_i are commutative matrices each of order 6^{i-1} with elements linear in $x_1, x_2, \cdots, x_n$. The determinants of U_n, V_n, W_n are each equal to f^k where $k = 2^{n-1} 3^{n-2}$. Hence, if sets of indeterminates (α), (β), (γ) each of order $p = 6^{n-1}$ exist such that

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