THE ABSOLUTE CONVERGENCE OF THE ALLIED SERIES OF A FOURIER SERIES

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I. Introduction

1. The present paper may be considered as a sequel to a recent paper on the absolute convergence of a Fourier series [3]. Some notations and terminology are the same and need not be explained here.

We suppose throughout that $f(t)$ is periodic with period 2π, and Lebesgue integrable in $(-\pi, \pi)$. We write

$$(1) \quad \varphi(t) = \tfrac{1}{2}\{f(x+t) + f(x-t)\}, \qquad \psi(t) = \tfrac{1}{2}\{f(x+t) - f(x-t)\}.$$

Let $\sum A_n(t)$ be the Fourier series of $f(t)$ and $\sum B_n(t)$ the allied series. Then

$$(2) \qquad \varphi(t) \sim \sum A_n(x) \cos nt, \qquad \psi(t) \sim \sum B_n(x) \sin nt.$$

In the paper [3], we have proved that the pair of conditions

$$(3) \qquad \int_0^\pi |\, d\varphi(t) \,| < \infty \quad \text{and} \quad \int_0^\pi |\, d(t\varphi'(t)) \,| < \infty$$

implies the absolute convergence of $\sum A_n(x)$, x being fixed. We prove in the present paper the corresponding theorem for the allied series: if

$$(4) \qquad \int_0^\pi \left| \frac{\psi(t)}{t} \right| dt < \infty$$

and

$$(5) \qquad \psi(\pi - 0) = 0,$$

then the pair of conditions

$$(6) \qquad \int_0^\pi |\, d\psi(t) \,| < \infty \quad \text{and} \quad \int_0^\pi |\, d(t\psi'(t)) \,| < \infty$$

involves the absolute convergence of $\sum B_n(x)$. The condition (5) is of course necessary for the absolute convergence. So also is the condition (4), as remarked by Bosanquet and Hyslop [2].

Denoting the modulus of continuity of $f(t)$ by $\omega(\delta)$, Salem [6] notices that Zygmund has established the following theorem [8]: for a function $f(t)$ of bounded variation, the condition

$$(7) \qquad \sum n^{-1}(\omega(n^{-1}))^{\frac{1}{2}} < \infty$$

Received January 25, 1946.