CYCLIC SUBSETS OF A GROUP

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1. **Introduction.** Let λ be an element of a group G whose elements are g_i. Let a be a positive integer. Consider the infinite set of quantities, not necessarily distinct,

$$\lambda, \lambda^a, \lambda^{a^2}, \cdots, \tag{1}$$

consisting of all the elements of G of the form λ^{a^i}, where $i = 0, 1, 2, 3$, etc., (1) will be called a *power set* of G of power a. Its *order*, n, is the number of distinct elements in it. The order of (1) is infinite if and only if the period of λ is infinite. Here it will be assumed G is finite and consequently all power sets of G are of finite order. For convenience λ^{a^i} will be written λ_i and (1) can be written in the form

$$\lambda_0 = \lambda, \qquad \lambda_1 = \lambda^a, \cdots, \lambda_i = \lambda^{a^i}. \tag{2}$$

Since G is of finite order there exist c and s with $s < c$ such that $\lambda_c = \lambda_s$. Let λ_c be the first element in (2) for which $\lambda_c = \lambda_s$ where $s < c$. Then (2) is of order c. The set

$$\lambda_s, \lambda_{s+1}, \lambda_{s+2}, \cdots, \lambda_c = \lambda_s \tag{3}$$

is an ordered subset of consecutive elements of (2). From its law of formation it is obvious that (2) consists of a finite number, possibly zero, of elements followed by subsets (3) repeated ad infinitum. Hence the power set (2) will be called periodic with *period* $c - s$.

If (2) contains the identity of G, its period is one and it is of little interest. Also (2) may not contain the identity but its order may exceed $c - s$. The remaining case in which $n = c - s$ will be investigated now. If $n = c - s$ then $\lambda_n = \lambda_0 = \lambda$ and the power set consists exclusively of an infinite sequence of the ordered subsets

$$\lambda = \lambda_0, \lambda_1, \lambda_2, \cdots, \lambda_{n-1}. \tag{4}$$

Let the operator R be defined by the equation $R(\lambda_i) = \lambda_{i+1}$ cyclically. The set (4) can be generated by starting with any element in it and repeatedly applying R. The subscripts form the additive cyclic group of order n. The set (4) where $\lambda_i = \lambda^{a^i}$ and $\lambda_n = \lambda_0 = \lambda$ will be known as a *cyclic subset* or J set of G and will be designated by $J\lambda_{an}$, where λ is its first element and will be referred to as a generator, a its *power* and n its *order*. When the identity of the first element is immaterial, (4) may be referred to as a J_{an} or simply a J set.

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