# FOURIER-WIENER TRANSFORMS OF ANALYTIC FUNCTIONALS

BY R. H. CAMERON AND W. T. MARTIN

1. **Introduction.** In the paper on the pages immediately preceding this paper, one of the present authors [1] has defined a Fourier-Wiener transform of a functional $F[x]$ as follows:

DEFINITION. Let $F[x] = F[x(\cdot)]$ be a functional which is defined throughout the space $K$ of complex continuous functions defined on $0 \leq t \leq 1$ which vanish at $t = 0$, and let $F$ possess the property that $F[x + iy]$ is a Wiener summable in $x$ over $C$ for each fixed $y(\cdot)$ in $K$. ($C$ is the subspace of all real functions in $K$.) Then the functional

$$G[y] = \int_C^w F[x + iy]\, d_w x \qquad (y \in K) \tag{1.1}$$

is called the *Fourier-Wiener transform* of $F[x]$.

In the present paper we consider three classes of functionals, and we show that if $F$ is a member of any one of these classes then the Fourier-Wiener transform $G[y]$ of $F[x]$ exists, belongs to the same class, has $F[-x]$ as its Fourier-Wiener transform, and satisfies the following form of Plancherel's relation

$$\int_C^w \left| F\left[\frac{x}{2^{\frac{1}{2}}}\right] \right|^2 d_w x = \int_C^w \left| G\left[\frac{y}{2^{\frac{1}{2}}}\right] \right|^2 d_w y. \tag{1.2}$$

The main theorem of the present paper is

THEOREM A. *Let $E_m$ be the class of functionals $F[x]$ which are mean continuous, "entire," and of mean exponential type; that is, let $E_m$ be the class of functionals satisfying the following three conditions:*

1°.
$$\lim_{n\to\infty} F[x^{(n)}] = F[x]$$

*holds for all $x$ and $x^{(n)}$ in $K$ for which*

$$\lim_{n\to\infty} \int_0^1 | x^{(n)}(t) - x(t) |^2 \, dt = 0;$$

2°. *$F[x + \lambda y]$ is an entire function of the complex variable $\lambda$ for all $x$ and $y$ in $K$; and*

Received May 19, 1945. The author wishes to thank Professors S. Bochner, R. H. Fox, and H. Samelson for various suggestions made during the writing of this paper.