SOME EXAMPLES OF FOURIER-WIENER TRANSFORMS OF ANALYTIC FUNCTIONALS

By R. H. Cameron

1. It is the purpose of this note to define a transform of a functional which is somewhat analogous to the Fourier transform of a function. The functionals will be defined as the space K of complex continuous functions $x(t)$ on the interval $0 \leq t \leq 1$ which vanish at $t = 0$, and the integrals will be Wiener integrals over the subspace C of all real functions in K. The usual factor e^{iux} which appears in the classical Fourier transform will not appear in the formula for the Fourier-Wiener transform because it is supplied by the exponentials which are inherent in the definition of a Wiener integral, and which therefore need not be explicitly written in the formula.

The transform is defined in accordance with the following definition.

Definition. *Let $F[x] \equiv F\,[x(\cdot)]$ be a functional which is defined throughout K and which possesses the property that $F\,[x + iy]$ is Wiener summable in x over C for each fixed $y(\cdot)$ in K. Then the functional*

$$G[y] = \int_C^w F[x + iy]\, d_w x \tag{1}$$

will be called the Fourier-Wiener transform of $F[x]$.

There exist wide classes of functionals $F[x]$ with the following property: The Fourier-Wiener transform $G[y]$ of $F[x]$ is such that $G[y - ix]$ is Wiener summable in y over C for each fixed $x(\cdot)$ in K and the reciprocal relation

$$F[x] = \int_C^w G[y - ix]\, d_w y \tag{2}$$

holds for every x in K. In the present note several examples are given of functionals $F[x]$ for which the reciprocal relation (2) holds. The author wishes to thank W. T. Martin for suggesting the use of Stieltjes integrals in these examples, and the use of the average value theorem for analytic functions in simplifying the proof of the reciprocal relationship. In the joint paper by Martin and the author which appears on the pages immediately following this note, three classes of functionals are defined, and it is shown that if $F[x]$ is a member of either of these classes, then the Fourier-Wiener transform $G[y]$ of $F[x]$ exists, belongs to the same class and has the Fourier-Wiener transform $F[-x]$.

We indicate the fact that $G[y]$ is the Fourier-Wiener transform of $F[x]$ by the notation

$$F[x] \rightarrow G[y]. \tag{3}$$

Received May 19, 1945.