A THEOREM OF B. SEGRE

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Recent results of B. Segre on lattice points in the star domain (see [2], [3] for the definition and properties of star domains)

$$-a < xy < b \qquad (a > 0, b > 0)$$

contain as a limiting case the following theorem (see [5; Theorems 2, 3]):

Theorem 1. *Let K be the point set*

$$-1 \leq xy < 0.$$

Then every lattice of determinant 1 *has at least one point in K, but a lattice of larger determinant need not have this property.*

This theorem is of interest since K *is not a star domain*; it is moreover nearly trivial that if H is any bounded subset of K, then lattices of arbitrarily small determinant exist which contain no points of H.

In this note, I give a short proof of Theorem 1 based on Mordell's method (for a short account, see [4]), and discuss further the connection with continued fractions.

1. **Proof of Theorem** 1. The parallelogram

$$\Pi: \qquad |x + y| \leq 1, \qquad |x - y| \leq 2$$

is of area 4; except for the triangle

$$T: \qquad x \geq 0, \qquad y \geq 0, \qquad x + y \leq 1$$

and the triangle $-T$ symmetrical to T in the origin $O = (0, 0)$, Π consists only of points of K.

Let now Λ be any lattice of determinant 1. Then, by Minkowski's theorem on linear forms, at least one point $P_0 = (x_0, y_0) \neq O$ of Λ lies in Π. The assertion is proved if P_0 belongs to K; so let us exclude this case. Then we may assume, without loss of generality, that P_0 lies in T.

Consider the straight line

$$L: \qquad x_0 y - y_0 x = 1.$$

Since Λ is of determinant 1, this line contains an infinity of lattice points, the distance between consecutive points being

$$\overline{OP}_0 = (x_0^2 + y_0^2)^{\frac{1}{2}}.$$

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